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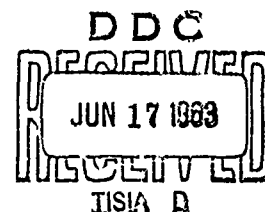
Final Report

COMMAND AND CONTROL SYSTEMS ANALYSIS

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FOREWORD

This document is the final report for contract AF 30(602)-2752 performed by the Military Products Research Department of Minneapolis Honeywell and published as Honeywell Document R-RD 1539-TR1.

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ABSTRACT

Systems analysis procedures are important for determining the proper allocation of information handling and decision making functions among the men in large man-machine organizations such as military command and control systems. Research is required for modeling and analyzing these functions and relating the sensitivities of these functions to the system criterion.

This report presents the steps of a generalized systems analysis procedure for command and control systems. The steps are then followed utilizing an existing SAGE Direction Control system as a vehicle for the study. Emphasis is placed upon modeling the human organization as a whole.

Industrial control system analogies are found to be useful to model the information handling and decision making functions of a command and control system. Bayesian formulations of the decision makers are utilized for relating decision errors and delays to a selected system criterion. Such formulations also indicate a criterion against which sequencing or priority rules in the various functions may be evaluated.

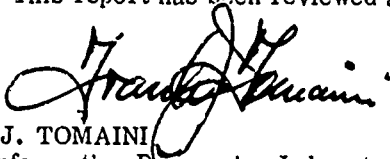
Directed graph techniques are suggested for developing the indirect relationships which can exist between the decision makers. Markov processes are utilized for modeling the decision making rates of a man-computer-display combination. Blocking effects between decision makers of the organization are shown to restrict the information processing rate capabilities of the organization. Combinatorial techniques are utilized for modeling the higher echelon control of the lower echelon decision makers.

The conclusions of the report are self-contained and recommended areas for future study.

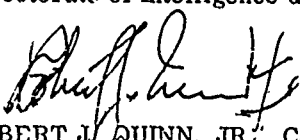
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COMMAND AND CONTROL SYSTEMS ANALYSIS

I. INTRODUCTION

During the past years many sophisticated automatic weapon systems have been analyzed, developed, and optimized ranging from submarine weapon systems to manned interceptor and guided missile systems.

The intelligent commitment of these weapon systems required up-to-date information of the enemy threat, the success of weapons committed, and knowledge of the status of our present forces.

These requirements have dictated the need for man-machine system organizations to perform high-speed communications, information handling, and decision functions for effective weapon control and utilization. These organizations are usually referred to as Command and Control systems or sometimes L-systems.

L-systems can be categorized into four basic types dependent upon the primary functions which they perform. These types are: (1) Command systems, e.g., SAC 465-L; (2) Control systems, e.g., SAGE 416-L; (3) Sensor systems, e.g., BMEWS 474-L; and (4) Support systems, e.g., Weather 433-L.

The numbers after each type refer to the more commonly known L-systems. This is by no means a complete list but is only meant to be representative. The report will be primarily concerned with the first two types of L-systems. Since many of the information handling and decision functions are similar in these two types, we will speak of them as Command and Control Systems in the remainder of the report.

Command and control systems are made up of the following subsystems: (1) Sensor and Effector subsystems; (2) External Communication subsystem; (3) Internal Communication subsystem; (4) Information Processing subsystem; (5) Decision subsystem; and (6) Off-line and On-line Control subsystems. These subsystems are shown in Figure 1. The command and control center is defined as being composed of all but the first two subsystems and will be the topic of the report. Each of these subsystems perform the information handling and decision functions shown in Figure 2 to achieve some defined on-line system criterion.

This criterion in a command and control system usually is some function of the information handling and decision speeds and accuracies. When the criterion is not

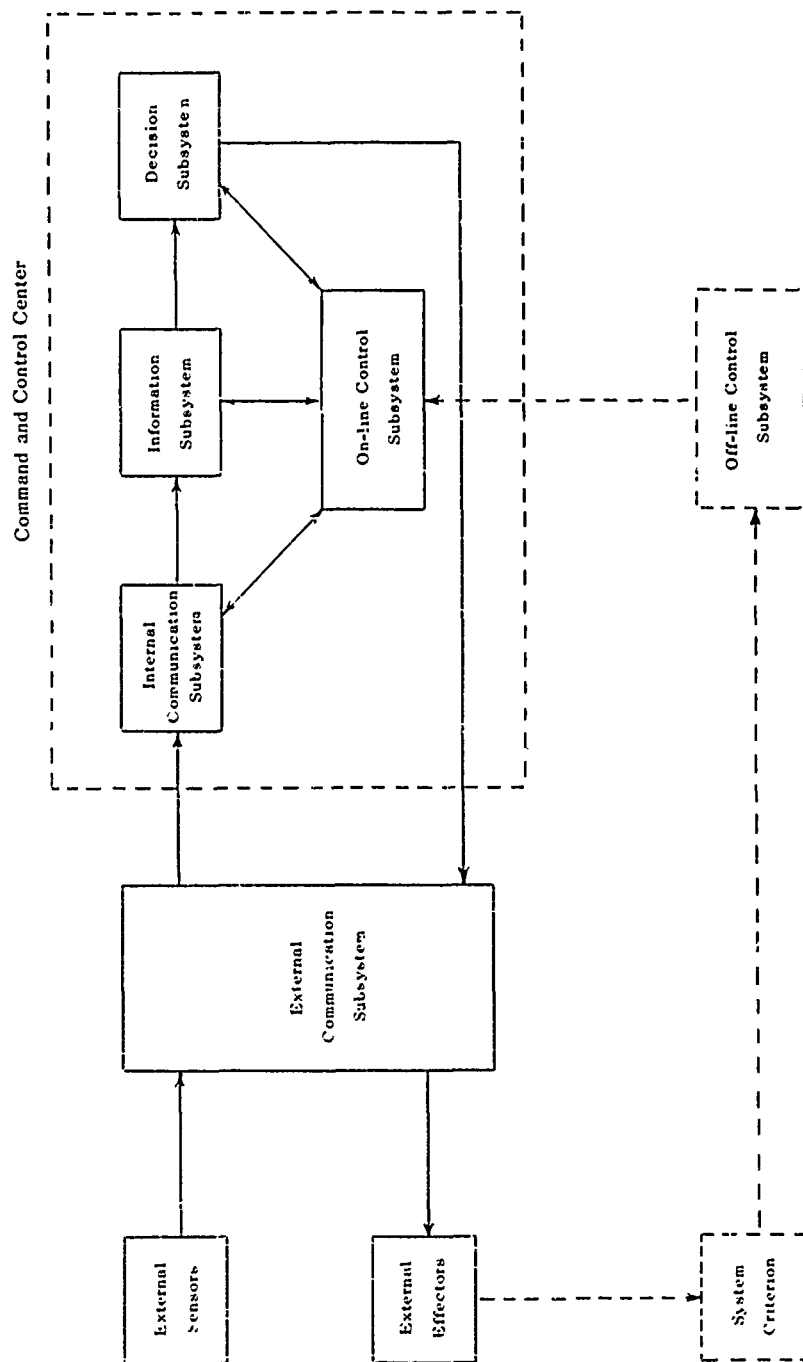


Figure 1. Command and Control System Diagram

Sensor Subsystems

Information Acquisition

Communication Subsystems

Information Dissemination

Information Subsystem

Information Storage

Information Transformation

Information Retrieval

Information Display

Decision Subsystem

Human Decisions

Effector Subsystem

Command Completion of Decision Subsystem

Information Dissemination

Control Subsystems

On-line Control of Communication,

Information, and Decision Subsystems

Off-line System Augmentation

Figure 2. Command and Control Subsystem Functions

met, the designs of the subsystems must be augmented or the on-line control subsystem operating procedures altered.

Command and control systems due to their size, complexity, interrelationships, dissimilar information handling functions, and complex human decision processes frequently cannot be analyzed or optimized by a systematic approach. Three approaches to the analysis and optimization of command and control systems are possible. These are:

- (1) War Gaming
- (2) Computer Simulation
- (3) Mathematical Analysis.

Wargaming involves the on-line simulation of the completed command and control system utilizing actual men and the designed subsystems. It can readily be seen that this approach to analysis can become quite costly.

Computer simulation presents the other alternative to analysis but must be preceded by extensive mathematical and system analysis before this becomes feasible and does not neglect many of the interrelationships.

Mathematical analysis has not been attempted in many cases due to the complexity of so many dissimilar subsystems and interrelationships. However, it can be seen that such a mathematical systems analysis is nevertheless a requirement for computer simulation approaches which in turn should reduce the present costs of wargaming approaches.

The purpose of this report is to develop a generalized system analysis procedure and mathematical models which can be utilized by command and control systems. The procedure emphasizes the systematic development of the on-line human information handling and decision functions of the system and the development of the human organizational structure.

The steps of the system analysis procedure are then demonstrated by utilizing an already existing command and control system as a vehicle for the mathematical models. The system utilized is a SAGE Direction Control Center.

Finally, the problem areas of each step of the system analysis procedure and recommendations for future study are indicated in the conclusions of the report.

II. SYSTEM ANALYSIS PHILOSOPHY AND PROCEDURE

The objective of system analysis is to define the interrelationships between the free system variables so that the system criterion can be achieved by manipulation of these free variables. The method of system analysis is to create abstract mathematical models of the actual physical system which can be used to study the behavior of the system when subject to various disturbances, control, and organizations. The system analysis procedure usually involves the following steps:

- (1) Definition of System Criterion
- (2) Development of Initial System Functions
- (3) Development of Abstract Models of Functional Blocks
- (4) Determination of Direct Relationships Between Functional Blocks and Organizational Structure
- (5) Determination of System Variables, Goals, and Indirect Relationships
- (6) Determination of Sensitivity of System Variables to System Criterion
- (7) Development of Functional Block and Interblock Control Flexibility
- (8) Refinement of Functional Blocks and Organizational Structure.

Figure 3 shows this sequence of steps.

The first step in the system analysis procedure is the definition of the system criterion. This criterion is sometimes difficult to define in command and control systems because it may be a complex function of many sub-criteria. These sub-criteria or goals may be either static or dynamic relations. That is, a minimum system design cost goal can be regarded as a static goal. Maximum accuracy or maximum speed of response can be treated as dynamic goals of the system. These dynamic goals relate to the on-line performance of the system after it has been designed.

Development of the goals of the functional blocks of the system must be accomplished so that the goals will not conflict with the overall system criterion. That is, in command and control systems the overall criterion is usually some function between speed of response (minimum time goal) and accuracy (minimum error goal). When a functional block's goal is defined it must take into account this time-error interrelationship. For example, a functional block may be a computer, a man, or a group of men. If accuracy is to be achieved in any one of these cases, information must be gathered before a computation or decision is made. The more information

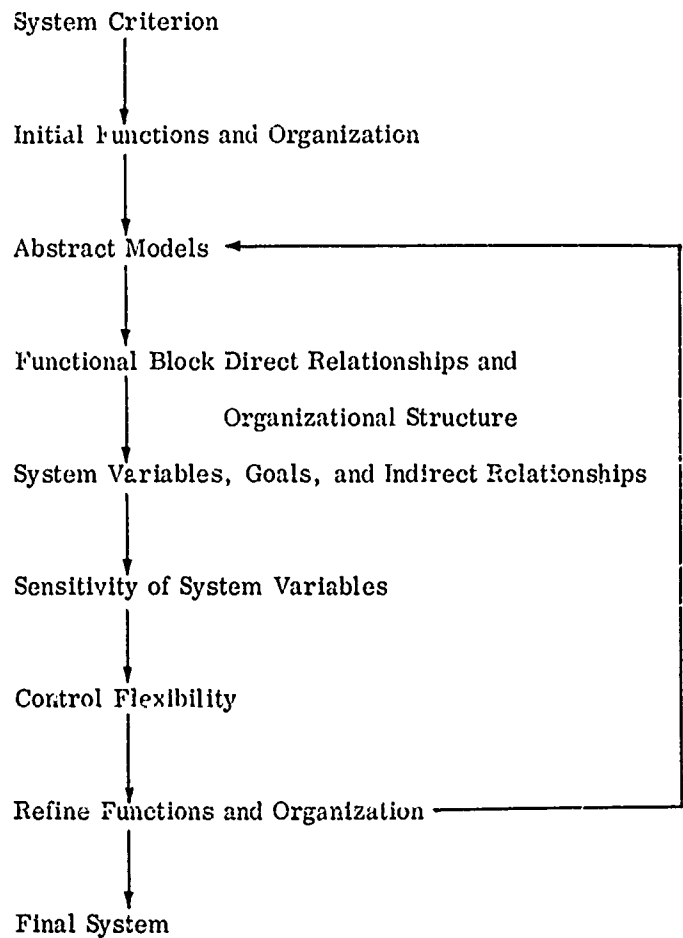


Figure 3. System Analysis Procedure

the higher the accuracy. However, the gathering of information usually involves time delays which are due either to communication delays or to noise conditions. Therefore, the problem becomes one of determining when to make the decision if the criterion is a function of both speed of response and accuracy.

Another problem in command and control systems regarding the system criterion is the interrelationships existing between different goals of the system. For example, one group in the system may have a goal which is dependent upon another separate group's goal. The problem then becomes one of determining the inter-group goal trade-off that is required to meet the overall system criterion.

Therefore, the criterion definition involves defining the goal trade-offs within a functional block and between functional blocks.

The second step is the definition of system functions and the organizational structure. The function definition is involved at the machine, man, group, and echelon levels of the system. These functions are usually always some form of information handling functions (Figure 2). In command and control systems the program functions of the computer and their sequence or priority logic is an important function at the system analysis level.

This defines which functions the computer will perform for the man and the time delay and accuracy of these functions. Again the time-accuracy trade-off is an important step when defining these program functions and logic.

The determination of which functions should be performed by men and machines is probably the most difficult in command and control systems because of the continual trade-offs which exist between the speed and accuracy goals.

As the input information rates to the system increase, the need for more capacity increases. This then requires more men and thus the groups and echelons begin to evolve. The problem now becomes that of distributing the functions between the men and groups. (This is discussed below in step 4).

The third step is the development of an abstract mathematical model that will integrate each of the functional blocks of the system in some common terminology. This pertains to the information transformations that take place in each functional block and to their inputs and outputs. These inputs and outputs must include all the noise factors as well as the information quantities. The abstract model is usually the "vehicle" which will integrate and relate the sensitivity relations of the various system variables to the one overall system criterion.

This is an important and difficult step with regard to command and control systems because of the many dissimilar functional blocks involved, such as men, displays, and computers. Each of these functional blocks and interrelationships usually cannot be described by the same mathematical disciplines. Therefore, a common terminology which will link the variables of each block together and finally relate them to the criterion is needed.

This is also an important step when analyzing the integration of a number of command and control systems with different functions and criteria. The overall model should then become a combination of smaller models all having the same terminology. The functional interrelationships should then become more apparent with this common terminology.

The fourth step in the procedure is the determination of the direct relationships required between the system's functional blocks. This defines the man-man, man-machine, and machine-machine direct relations. As stated above, as information handling functions increase in number and rate, increased capacity is required. This requires more machines, men, groups, and echelons. Each of these functional blocks must then be interrelated in a manner that will assure the system criterion being achieved.

Communication links and routes must be determined between these functional blocks for the passage of decisions, information, control, and commands.

The second set of interrelationships is the information transformation sequences of the system. If the functional block is a computer, this means the determination of the proper program sequences and priorities. If the functional blocks involve men, then the proper sequences by which information is processed by and collected from the men must be determined. This step usually defines the precedence relations between the information transformation sequences required before system outputs result. These outputs may refer to decisions, commands, control, and/or information.

The third set of interrelationships refer to the decision, command, and control authorities existing between the men, groups, and echelons of the organization. In command and control systems this becomes a problem of determining how much authority can be given to the higher and lower echelons. If no authority is given to the lower echelons then speed has to be sacrificed. This is because of the back and forth communication required between the echelons before a decision or information

transformation can be completed by the lower echelon and accepted by the higher echelon. However, if too much authority is given to the lower echelons, then the control or decisions may be made without knowledge of the overall system operation. In this case, the speed advantage (of not having to communicate with the higher echelon) can be outweighed by the possible ineffectiveness of the decision made with insufficient knowledge of the present system state.

Therefore, the direct relationships of the organizational structure can directly affect the time-effectiveness criterion of a command and control system and become an important step in the procedure.

The fifth step in the system analysis procedure is one of the most difficult. This is the determination of the system variables, goals, and indirect relationships. The functional blocks are defined in step two. The direct relationships are established in step four. The problem now becomes one of first determining all of the system variables.

These variables pertain to the functional blocks and the direct relationships. That is, a variable of a man (functional block) may be his decision time delay. A variable of a communication link (direct relationship) between two functional blocks may be its information rate. These are free or controllable variables and can be varied by on-line control or off-line design augmentation. Uncontrollable variables pertain usually to noise. In command and control systems, noise is defined as any deviation from a norm. For example, wrong communication paths, erroneous decision, delayed decisions, and delayed information may all represent noise to the system when uncontrollable.

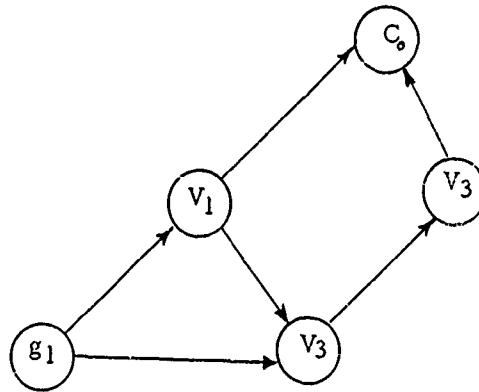
After the variables are defined, all of the sub-criteria or goals of the system's functional blocks must also be defined. In the case of a computer functional block, this refers to the time and accuracy requirements (goals) of each program in the computer. In the case of a group of men in the organization, this will define the objective function (goal) of their information handling and decision process. This goal may also be some trade-off between decision speed and accuracy.

After the variables and goals are defined, the final part of step five in the procedure is the determination of all the indirect relationships existing between the variables, between the goals, and between the variables and goals of the system. Detection of the indirect relations can then sometimes proceed logically if the direct relations are found. The problem in command and control systems is the definition

of the direct relations between the variables. This is because of the dissimilarities existing between the variables of the function blocks and the variables involved in the direct relationships. The purpose of the abstract model development (step 3) between steps (2) and (4) becomes one of aiding both the direct relationship development of the system (step 4) and the determination of the indirect relationships between the system variables (step 5).

After the goal-goal, variable-variable and goal-variable direct and indirect relationships have been determined, they are finally linked with the system criterion. That is, each goal of a group in the system will have some functional relationship to the overall system criterion which must be defined in some manner.

When step five in the procedure is completed, a graph showing the system direct relationships may look as shown below.

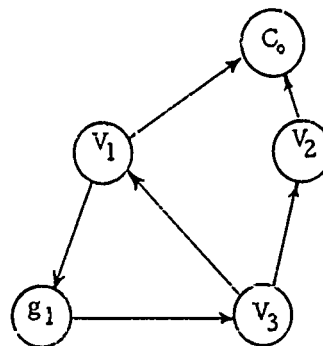


The V_i 's may represent the variables of the functional blocks. The g_i 's may represent goals of these functional blocks and C_0 may represent the system criterion. The arrows between the nodes of this graph represent some direct relationship or effect existing between the nodes. Here the direct relationships such as $g_1 \rightarrow V_1 \rightarrow V_2 \rightarrow C_0$ can be readily seen. When the number of nodes (variables and goals) in a system become large, such visual detection becomes impossible and must be performed logically by matrix techniques. It should be noted that such techniques merely locate the indirect relationships in the system but say nothing about their functional relationships.

The sixth step in the system analysis procedure is the development of the sensitivities of the system variables and goals to the system criterion. The sensitivity

analysis must relate, in some quantitative or qualitative manner, the variables and goal relationships to the system criterion.

The sensitivity analysis is by far the most difficult step in the procedure for many reasons. First, there is the problem of dissimilarities existing between the functional block variables which may prevent analytical relationships from being developed. Second, there may exist no analytical techniques to develop quantitative relations. Third, even if analytical techniques can be developed for each direct relationship, there exists the problem of linking them together and relating the indirect relationships to the system criterion. For example, to relate how variable V_1 indirectly affects the system criterion C_0 via V_3 and V_2 may involve a number of different analytical techniques such as decision theory, queueing theory and sequencing theory. The fourth problem relates to gathering of experimental data on the system performance. In command and control systems such data may not exist to perform any sensitivity analysis until the actual system has been built and functioned. This becomes a costly process. The problem then becomes one of collecting quantitative data from other systems and experiments with men and relating it to the newly conceived system and criterion. The fifth problem becomes one of determining the sensitivities of the system variables when cycles exist in the indirect relationships. For example, consider the following diagram:



Variable V_1 affects goal g_1 which affects variable V_3 which in turn affects variable V_1 . The problem now exists in relating the sensitivity of the nodes of this cycle to the criterion C_0 . We can sometimes detect such loops systematically but how to relate them to the criterion can become difficult. The sixth problem is one of defining the proper functional relations of the goals to the system criterion and determining the proper trade-offs between goals when they are interrelated. For example,

the system criterion may be some function of a minimum time and a minimum error goal of a group. If these two goals are interrelated in some manner the proper trade-off must be determined. This can only be achieved by knowledge of the sensitivities of each of these goals to the criterion via their effects on the functional block variables.

The seventh step in the system analysis procedure is the detection or development of any control flexibility within the functional blocks and the control rules between the functional blocks. This step utilizes the above sensitivity analysis to exploit the dynamic variables of the system. Dynamic variables refer to the operating procedures of the system. Static variables refer to the design characteristics and cannot be changed during on-line operation of the system. The dynamic variables are usually always exploited before the static variables are changed.

Dynamic variables in command and control systems usually refer to communication routes between the functional blocks, information processing sequences, information processing time delays and errors, and decision time delays and errors. Control within the functional blocks usually depends on the time-accuracy goal trade-offs developed from the sensitivity analysis. For example, it may take the form of determining the time a man should take gathering information before he makes a decision.

Control between the functional blocks involves any switching, sequencing, and scheduling functions. At the computer program level this may involve the on-line control of the program sequences in the central computer. At the echelon or group levels of system, it may involve the control of the information processing and/or decision sequences of the men in the system. That is, it may involve the determination of possible higher echelon control of the lower echelons or the control existing between groups at the same echelon.

Development and detection of any of the above control possibilities is important in command and control systems for the purpose of determining how the system can react in the case of overloads. It also is important for detecting any modular flexibility of the functional blocks for future system augmentation and reorganization potentials.

The final step in the system analysis procedure is the refinement of the functional blocks and organizational structure of the system. This step utilizes the relationships developed from the sensitivity analysis to change the static system variables

to meet the system criterion. The control analysis above exploits all the possible on-line flexibility of the system to meet the criterion. When this dynamic control cannot achieve the criterion, the only remaining step is that of introducing static functional and organizational changes.

Static functional changes may involve redesign of the functional blocks such as the computers and displays. Static organizational changes refer to the direct relationships in step 4 of the procedure. These may involve new or different communication paths, information processing sequences, decision, command, and control authorities being established.

Constraints within the functional blocks and precedence relationships between the functional blocks may exist such that the above changes are not possible. In this case, the addition of parallel channels is the only alternative left. This may require additional computers or more men at the echelon and group levels of the system.

When this last step has been performed, the system analysis cycle may repeat itself. This is because enough variables may have been changed to make the functional relationships developed in the sensitivity analysis no longer applicable to relating the newly augmented functional blocks and organization to the system criterion. Once the criterion is satisfied the system is defined.

In summary, some of the questions that arise in the system analysis of command and control systems are:

- (1) How can the system best be modeled?
- (2) What mathematics can be utilized for the manipulation of large number of variables and their interrelationships?
- (3) How can two or more systems be compared and/or evaluated?
- (4) How can the functions required of the system best be distributed between the men and machines?
- (5) What kind of machines provide optimum performance?
- (6) How should men and machines be organized and coordinated?
- (7) How can the effectiveness of the system be related to the effectiveness of its components?
- (8) What can be done to develop the full potential of the capacity of the system in case of overloads?
- (9) What functions can be relaxed and how do their sensitivities affect the final system goal?
- (10) Can groups in a system operate with independent goals?
- (11) What models are required to model the man-machine interrelationships?

Probably the simplest definition of system analysis, in the field of command and control systems, is the answering of the question: Who, does what, when, and how? These answers must come from some systematic approach such as the steps shown in Figure 3.

III. OBJECTIVES AND APPROACH

The previous section presented the steps for a general system analysis procedure of command and control systems. Some of the problem areas associated with such analysis were briefly discussed. Most of the problem areas are related to the determination of the proper information handling and decision making functions for the men, groups, and echelons of the organization. These functions must be distributed among the men and controlled by the men in a manner that achieves the on-line system criterion. The main problem then becomes that of determining a systematic procedure for determining the optimum distribution and control of these information handling and decision making functions.

A SAGE Direction Control Center was selected as a vehicle for the study. It should be noted that the study was not intended to be another attempt to optimize SAGE, but rather to use SAGE to investigate the feasibility of developing a systematic system analysis procedure. There were two basic reasons for selecting SAGE. First, the information handling and decision making functions of SAGE appeared to be typical of what might be expected in other command and control systems. Second, the problem areas of the systems analysis procedure would readily become apparent from an already operating system.

The objectives of the study which follows were:

- (1) To develop a generalized abstract model of the functional blocks of the SAGE Direction Center which could be utilized by other command and control systems (Section VI-A).
- (2) To develop models for the information handling and decision making functions of the men in the organization. These models should emphasize the variables of these functions which are useful for developing the organizational structure and sensitivity analysis (Section VI-B).
- (3) To develop the organizational structure of the men in the system in a manner to achieve the system criterion. This defines the control possibilities, information transformation sequences, decision authorities, and communication paths (Section VI-B).
- (4) To develop an approach for detecting the indirect relationships which exist between the variables and goals of the system's man, group, and echelon functional blocks (Section VII).

- (5) To develop a sensitivity analysis approach for relating the information handling and decision making variables of the men to the overall system criterion. The sensitivity analysis should be developed for purposes of suggesting functional and control changes to meet the system criterion (Section VIII).
- (6) To develop an on-line control model of the information handling and decision making functions of the men in the lower echelon. This model should exploit the feasibility and requirements of the higher echelons for controlling the functions of the lower echelons (Section IX).

The sections where each of the above objectives are discussed are indicated. The approach of the report will be to follow the system analysis procedure shown in Figure 3.

A simplified system criterion for the SAGE system is developed in Section IV and will be utilized throughout the report. The basic SAGE system functions and organization are presented in Section V.

The abstract model development utilizes many of the analogies of an industrial control system for modeling the information handling and control and decision making functions of a command and control system such as SAGE. The abstract model is then applied to modeling the decision making functions of the basic men involved in the SAGE system. These models emphasize decision error and time delay variables utilizing decision theory concepts. The abstract model development and final system organizational structure is presented in Section VI.

The indirect relationships which affect the information handling and decision making functions of the men in the SAGE system are investigated to a limited extent by directed graph (digraph) and matrix techniques in Section VII.

The sensitivity analysis relates the decision error and delay variables of the men in the system to the overall system criterion defined in Section IV. Loss functions are developed for decision errors and delays. The sensitivity analysis then discusses the minimization of the overall system criterion loss function by developing decision delay and error constraints (goals) for the men. To perform such analysis, decision theory and Markov processes were utilized in Section VIII.

The on-line control of the decision making and information handling sequences of the lower echelon by the higher echelon was investigated utilizing combinatorial analysis models. These models were developed for investigating minimum time de-

cision sequences involving a number of men in an organization such as SAGE. This is discussed in Section IX of the report.

Finally, the refined system functional and organization possibilities are discussed in Section X. The conclusion and future study recommendations are self contained and are presented in Section XI.

IV. SYSTEM CRITERION

The first step in the system analysis procedure is the definition of the system criterion and any goals (sub-criteria) which relate to the overall criterion.

Criterion. The system is defined as a defensive system. The criterion will be defined as a very simplified strategic situation where the purpose of the system is assumed to be the defense of one city. Interception resources for this purpose consist of two tactical interceptors.

Since the objective of the system is to protect one city, it is considered that there is a loss of unity when the enemy carries out a successful attack on the city. There is also a loss associated with the unintentional destruction of a friendly airborne object but this loss is difficult to fix quantitatively because its relationship to the defense of the city is complex and not singular and possibly even nonexistent.

Goals. There is defined three functional blocks which perform the information handling and decision functions to achieve the above criterion. These are defined as the surveillance, identification, and weapons assignment groups.

It is evident from the defined loss criterion that each of these groups will have a minimum error goal and a minimum time goal. No model of performance can be developed for these functional blocks without reference to these goals and how the goals relate to the criterion.

For example, consider the surveillance function. In this case the objective is to acquire information about all airborne objects (tracks) detected by radar. It is evident that if it is attempted to establish the coordinates for a particular track with undue haste, the information transmitted to the rest of the system might be of a quality too inferior for effective use. On the other hand, if a great deal of time is devoted to a particular track in order to determine its coordinates accurately, the information may be presented to the weapons assignment group too late for effective use.

Therefore, some sort of trade-off between a minimum error goal and a minimum time delay goal is involved at several points in SAGE. The problem of mathematical modeling is equivalent to the establishment of loss functions which relate the error-time delay trade-offs.

V. SYSTEM FUNCTION AND INITIAL ORGANIZATION

Once the system criterion and functional goals are established, the functional blocks must be clearly defined and the initial organizational structure established. This establishes the basic information handling sequences between the functional blocks and the echelon and group levels of the system.

Since the vehicle for the study is an already existing system, we can only briefly redefine its functional blocks and organization and then proceed with the system analysis procedure. The following descriptive definition of the SAGE Direction Control Center is covered fully in reference (1):

"A SAGE Direction Control Center receives digitally-coded data automatically and continuously from search radars, gap-filler radars, and height finder radars over voice-bandwidth communication circuits. Data on flight plans, weapons status, weather, and aircraft tracks are received, respectively, from the Air Movements Identification Service (AMIS), weapons bases, USAF weather service, Ground Observer Corps, and Airborne Early Warning and Picket Ships over teletype and voice-telephone circuits. Similarly, data from the direction center are transmitted in digitally-coded form over voice-bandwidth communication circuits to ground-to-air data link system, to weapons bases, to adjacent direction centers and to command levels. Data to other users are transmitted over automatic teletype circuits."

Figure 4 is a diagram of the system sensor inputs and the outputs to the system effectors. The command and adjacent control center can be either a sensor or an effector system depending upon the particular situation.

The functional blocks of the system can be separated into four basic information handling and decision making functions. These are defined as the surveillance, identification, weapons assignment, and battle staff groups (Figure 5).

The information handling functions of the surveillance group involve the processing of radar returns to determine which radar returns represent noise and which represent actual airborne objects (defined as tracks). The decision making functions of this group involve first the track or no track decision, and second, the decision

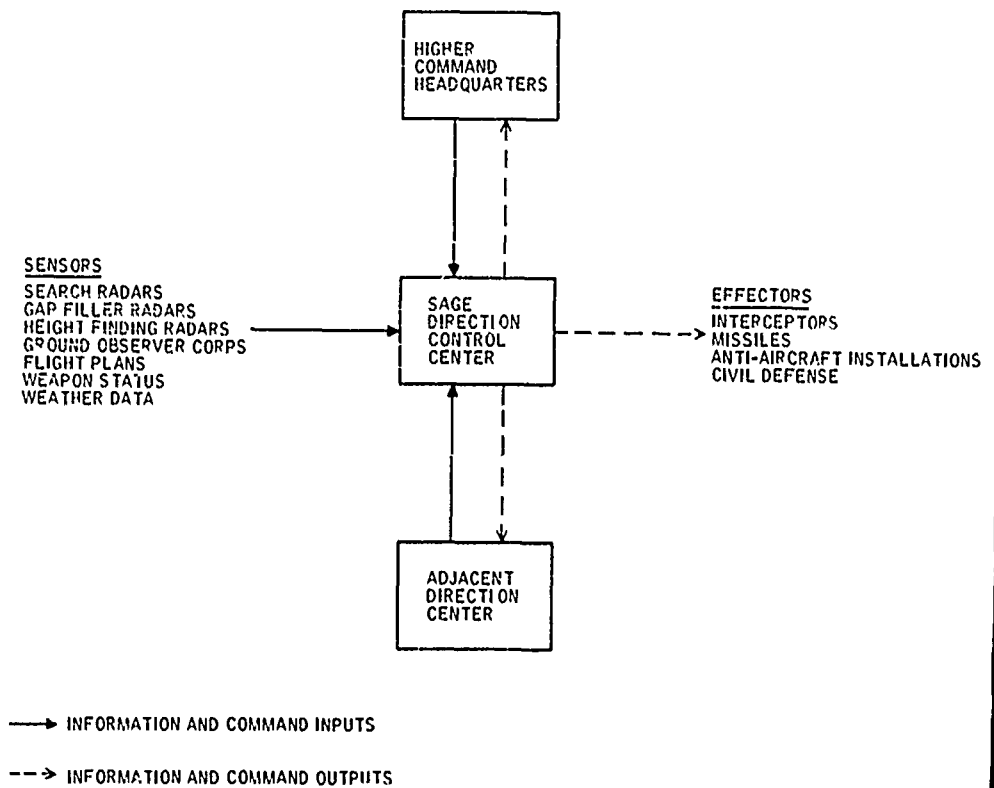


Figure 4. SAGE Direction Control Center Inputs and Outputs

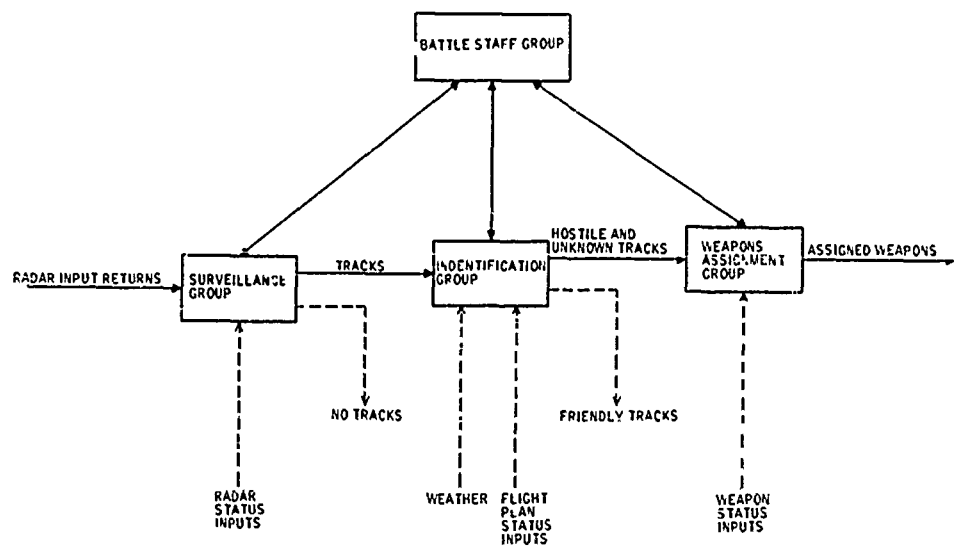


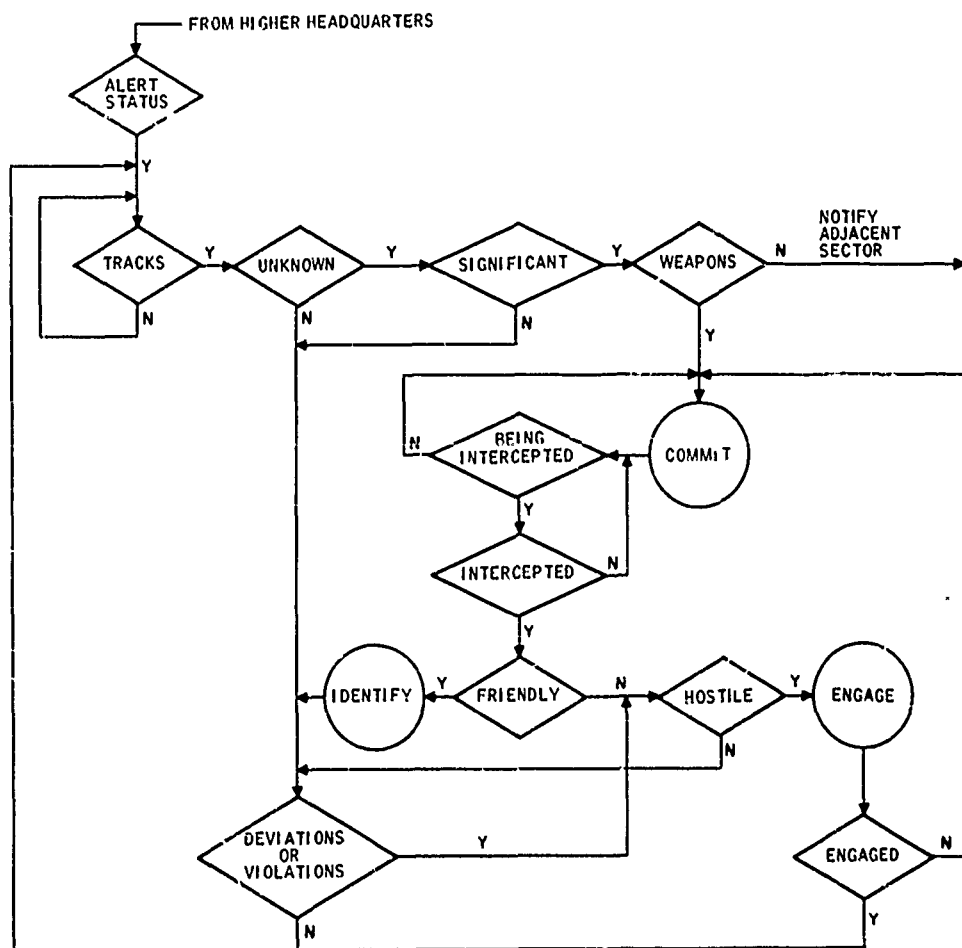
Figure 5. SAGE Functional Block Diagram

as to the coordinates of the tracks with respect to their position, velocity, direction, and height coordinates. When these decisions are made, this track information is then sent to the identification group. The radar status inputs to this group represent all the known noise conditions and operating status information about the radar feeding the system.

The information handling function of the identification group represents one of correlating tracks received from surveillance with logged flight plans of friendly aircraft. The flight plan status inputs represent the number of flight plans filed in the various geographical regions of responsibility of the system. These flight plans give the predicted coordinates of the friendly aircraft as a function of time. The decision making functions of this group involve the determination of friendly, hostile, and unknown tracks. Unknown and hostile track information is sent to the weapons assignment group.

The information handling functions of the weapons assignment group involve assignment and control functions. The weapon status inputs represent the numbers and operating status information on all the weapons. The decision making functions of this group involve first the determination of the proper assignment or allocation of weapons against the track, and second, the decision of destruction or visual identification. Finally, the intercept coordinates have to be established before weapon control can take place.

The functions of the battle staff group are assumed to be the coordination and control of and between the above three groups. The function of this group and the operation of the system can best be described by the decision flow diagram shown in Figure 6.



Y - YES DECISION
N - NO DECISION

Figure 6. Assumed System Decision Flow Diagram

VI. ABSTRACT MODELS OF FUNCTIONAL BLOCKS

The abstract model development should be utilized to integrate the functional blocks of the system in some common terminology. This pertains to the information handling and decision functions of the men and machines. The abstract model should develop a common terminology for these functions which will more clearly define the sensitivities of the variables to the system criterion. The level of this abstract model development is dependent upon the goals of each of the functional blocks and the system criterion. These goals relate to time and error parameters of each group. The abstract model development should be one which will aid the development of the direct relationships between the functional blocks, the determination of pertinent variables, and the sensitivity analysis, while being general enough to be applied to other command and control systems.

A. INDUSTRIAL CONTROL SYSTEM MODEL

This rather broad requirement for command and control systems is a problem when considering the natures of the many dissimilar functional blocks and information handling and decision functions.

It was found in studying the SAGE system that the functions of an industrial control system yield a suitable framework for such an abstract model development. This was particularly true when considering the modeling of the many dissimilar information quantities, noise characteristics, and information handling, control and decision functions.

This section will model the functional blocks of a command and control system center (Figure 1) such as SAGE in the terminology of an industrial control system. Figure 7 shows some of the analogies to be discussed in this section.

1. Material Information Inputs:

These inputs represent all the inputs to the system from the sensor systems or the external environment. The material information can have three classes:

- (1) Information Material
- (2) Command Material
- (3) Noise Material.

<u>Industrial Control System</u>	<u>SAGE Direction Control Center</u>	<u>General Command and Control Systems</u>
(1) Input Commodity Orders	(1) Input Stimuli (Radar Returns)	(1) Input Information and Commands
(2) Commodities	(2) Airborne Tracks	(2) Information Patterns
(3) Material	(3) Input Track Information	(3) Input Pattern Information
(4) Commodity Material State	(4) Track: Information State	(4) Pattern Information State
(5) Machines	(5) Computer, Displays	(5) Computer, Displays
(6) Man-Machine Combinations	(6) Man-Computer-Display Modules	(6) Man-Computer-Display Modules
(7) Output Finished Commodities	(7) Output Decisions and Weapon Commands	(7) Output Decisions, Information and Commands to Effectors

Figure 7. Analogies Between Industrial and Military Command and Control Systems

The information material can be such things as numerical data, voice communication, and radar returns from a radar site. Command material contains information but is separated because of its direct effect on the control processes of the system. Noise material represents the errors in the information and command material inputs. They are separated discreetly from the first two material inputs because they can enter the system independently. For example, erroneous or extraneous data could represent noise material. The erroneous noise material could be dependent upon the information material. The extraneous data represents noise material completely independent of any information material.

The material inputs can be described by their geographical location and their physical form. For example, a radar return entering the SAGE Direction Center comes from the geographical location of the radar site. The physical form is an electrical pulse. It is then transformed at the center into position information by a human observer.

The uncontrollable variables (or noise) of material information inputs are their delays, errors, and extraneous inputs. Noise in a command and control system is defined as any deviation from accuracy.

The controllable (or free) variables are associated with delays and errors. These can be controlled by alternate communication routing to the center and by code lengths.

2. Machine Elements:

The machine elements are all the non-human elements of the system which are capable of any kind of information handling function. These elements can represent such things as displays, computers, programs of the computer, and machines.

It is important to define the level at which the system analysis procedure must go in defining the machine elements. In a command and control system such as SAGE, with the minimum error and minimum time delay goals, this level appears to be the program level of the computer. That is, only the time delay and error of the program element, that processes the human element's information request, need be defined.

The programs of a command and control system computer become no more than the variety of machines of an industrial control system all placed in a

central location. These programs must still be time shared among the human elements of the system. In an industrial control system the machines must also be time shared among the commodities they are built to produce. This time sharing analogy is an important point in the system analysis as will be pointed out later.

The controllable variables of the machine elements are capacity, error, and time delays associated with their information handling processes. These controllable variables can be either dynamic or static variables. Dynamic refers to on-line control of the delays and errors. Static refers to off-line design changes of the machine elements which later alter these delays and errors.

The uncontrollable variables (or noise) also relate to capacity, errors, and time delays. For example, if the processing capacity of a computer is reached, some information requests of the human elements must be either delayed or rejected. Another alternative is to process all the requests at reduced accuracy (increased error). Thus, if system capacity is reached, many different noise effects can result.

3. Module Elements:

The module elements are defined as any integrated human element-machine element combination that cannot function alone when performing an information handling and/or decision function. In industrial control systems, this module represents a man-machine combination. In command and control systems, this module will be defined as a man-computer program-console display element combination.

The machine elements (computer program and console display) of the module usually perform the information transformation processes on input material information to the module. The human element performs a decision process which changes the state of some information pattern (see below).

The SAGE system is characterized by many of these module elements as described in the next section.

The variables of the module element are the same machine variables discussed earlier. In addition, there now exists the variables of the man's decision process.

The variables of the decision process are again the time delay and error variables. These can be either controllable or uncontrollable.

They are controllable when other human elements affect the decision process of the module or the particular human element affects its own decision process by some constraint. They are uncontrollable when noise factors (erroneous and extraneous information, delays, etc.) affect the module's function..

The above represent the dynamic variables. Again, the static variables relate to the design characteristics of the machine elements of the module.

4. Commodity Information:

Commodity information represents the most important aspect of command and control systems. It also represents one of the primary reasons for utilizing the industrial control system as the abstract model.

Commodity information is defined to represent the information patterns of the system. Commodity information represents the relationships between the material input information and the module element.

A commodity at the beginning of an industrial control system is nothing more than an initial order from a potential customer (Figure 7). It can be thought of as being in the "concept state" at this time. As this order (commodity) flows through the man-machine elements of the system, its material state is transformed. This is performed by the addition of input material to it. Each time a material element is added to the commodity by some machine process, its own material state is transformed to a higher and higher degree of formulation. Finally, it is completed and is sent out of the system to the potential customer.

In a command and control system, such as SAGE, the commodity represents a track of an airborne object. At the beginning of the system this track (commodity) is also in a "concept state." That is, the radar returns have not yet been transformed into a decision that it is or is not a track. As more and more returns (material information) are transformed by the computer program and display elements, a decision by the human is made. When the decision is made that the returns represent a track, the information state of the track changes from the "concept state" to the "position information state." The information state of the track (commodity) is said to be transformed by the man-computer-display module element. The material information required for this transformation were the radar returns.

Therefore, a commodity information state represents the information state of the information pattern for which the input material information was intended to develop.

An enemy raid is an information pattern which can be defined as commodity information of a higher level. That is, each of the information states of the tracks make up the raid information pattern. These now lower level information patterns (tracks) represent input material information to the raid pattern which is now higher level commodity information with respect to the tracks. Therefore, commodity information or information patterns can exist at different levels and are relative.

In general, commodity information is usually any information pattern which evolves as the result of combinations of input material information. The detection of and reaction to such information patterns are the main purposes of the centralized command and control system concept.

The variables of commodity information (information patterns) are their information state errors, time delays, and information state transformation sequences by the modules.

These can be controllable or uncontrollable variables. Controllable sequences may mean that a commodity (track) is allocated to a number of modules to process. Uncontrollable may mean a wrong sequence of commodity (track) information state transformations by the modules. This then represents noise to the system. That is, a wrong sequence can be treated as a deviation from accuracy and thus be noise.

5. Material and Commodity Information Transformations:

There are two information processes that are performed in command and control systems. The material information transformation is the process that "prepares" the material information to be used for the commodity information transformations. The material information transformation can be performed by any of the man, machine, or module elements. The commodity information transformation can be performed only by a human or module element.

The commodity information transformation is defined as a decision process that involves transforming the information state of the commodity information. For example, radar returns can be thought of as material information which undergoes a transformation process by a computer element (computer program). This process prepares the radar returns for another material information state transformation by the display element. At this time the material information (radar return) is ready to be applied to the commodity information. The commodity information is

a possible airborne track. The man then performs the commodity information state transformation by utilizing the transformed material information (radar returns) to make a decision. When he makes the track decision, the commodity information (track) undergoes an information state transformation from the "possible state" to the "position information state."

It is important to note that each time a decision is made (commodity information transformation), material information is usually required before another decision or transformation can be made. Thus, as commodity information flows through the module elements of a command and control system such as SAGE, it usually requires an alternate succession of material information state transformations before each commodity information transformation (decision). Figures 8a and 8b show the analogies between the commodity and material state transformations in an industrial control system and the commodity information and material information state transformations in a command and control system such as SAGE.

The variables of the information transformation processes are their time delays and errors. For example, a material information state transformation may require a number of iterations in a computational process by a computer. The time delay of the transformation process can be reduced by reducing the number of iterations. However, this may result in a large error. On the other hand, the error can be reduced by performing additional iterations at the expense of a time delay.

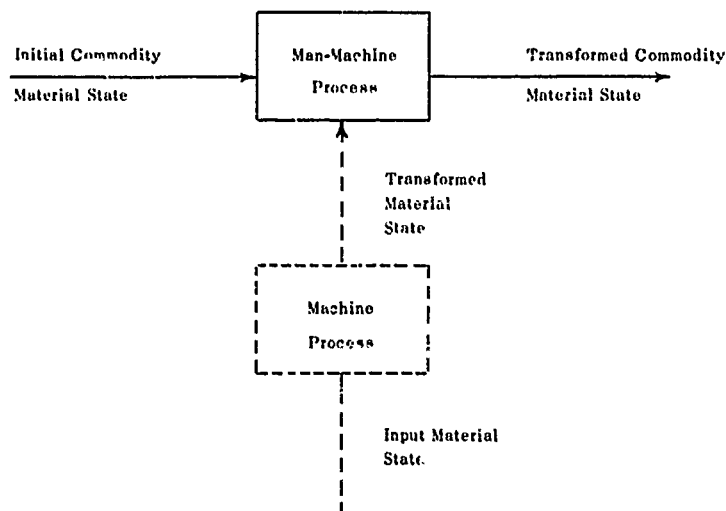


Figure 8a. Commodity Material State Transformation Process

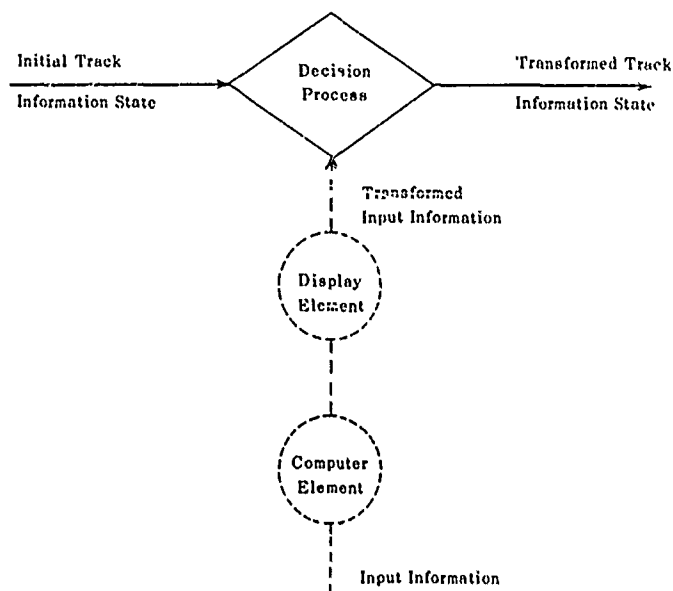


Figure 8b. Track Information State Transformation Process

Therefore, we see here the interrelationships existing between the time delay and error goals of a material information transformation process.

A commodity information state transformation can be varied in the same manner. However, now it involves a human decision process. For example, a man may spend time collecting material information so that his decision process involving a commodity information state transformation will be correct (minimum error goal). He sacrifices a time delay for smaller probable error. On the other hand he could make a faster decision (minimum time goal) on less information and sacrifice a higher probable error.

These error-time delay trade-offs are characteristic of the information handling and decision processes in command and control systems such as SAGE.

In both the material and commodity information transformation processes, their trade-offs and control can only be made when the system criterion is defined quantitatively.

Again, these error and time delay variables can be classified as controllable or uncontrollable and static or dynamic.

6. Information Subsystem:

With the above definitions, the information subsystem in Figure 1 can now be defined. It is defined as consisting of all the machine elements of the com-

mand and control center which perform the material and commodity information handling and transformation processes.

The variables of the information subsystem are its information storage capacity, speed, errors, information display rates and capacity, and the logic of its information communication and transformation sequences. These can be controllable or uncontrollable and static or dynamic variables.

7. Decision Subsystem:

The decision subsystem consists of all the human and/or module elements which perform any material and/or commodity information state transformations.

The variables of this subsystem usually refer to the decision authorities of the human and module elements and the organizational structure of the groups and echelons composed of these elements.

8. Communication Subsystem:

The communication subsystem internal to the command and control center in Figure 1 provides the paths between all the functional blocks of the system. These functional blocks can be defined as the humans, machines, modules, groups, and echelons. The paths then carry all the material and commodity information, decisions, control, and commands between these functional blocks. These paths are made up of links and routes. A link is defined as a direct path between two functional blocks. A route is defined as a succession of links (more than one) between two functional blocks.

The variables of this subsystem are its link and route information rates and its link and route organizational structure. The organizational structure defines all possible paths between the functional blocks.

9. On-line Control Subsystem:

The on-line control subsystem is made up of any man, machine, and/or module elements that can control the material and/or commodity information transformation and handling variables. This control in command and control systems usually can take two forms. First, the delay or error of the information transfor-

mation process can be controlled. Second, it may involve the control of the communication routes and transformation sequences of the material and commodity information.

This is the most important subsystem of the command and control center. Again, it is another reason for utilizing the industrial control system analogies for defining the functions of the center. These analogies or abstract models make it easy for relating the control of the information handling and transformation variables to the group goals and finally to the overall system criterion.

For example, in an industrial control system the manager usually has a number of different types of commodities to produce on a given number of machines. If m equals the number of different commodities and n equals the number of machines, there exist $(m!)^n$ possible sequences by which to produce these commodities, neglecting precedence relationships. There also exists the problem of what material to add to the commodities. If there is no time deadline on the production of a commodity, he may wait for higher grade material to increase the probability of selling his commodity. If there is a deadline, he must make the choice of immediately adding available lower grade material or waiting for higher grade material. His choice is then dependent upon the probabilities of the customer buying the late and higher grade or early and poorer grade commodity. Since this same problem exists on all the m different commodities, he also has the above problem of determining the best sequences through his n machines to satisfy all the different customers.

In command and control systems, such as SAGE, analogous situations can arise with the information patterns (commodity information) of the system. For example, the higher echelons (managers) of SAGE can be thought of as controlling the information handling and decision processes of the lower echelons by setting time delay and/or accuracy requirements on these processes. The lower echelons may be unable to gather enough material information within some time deadline, due to noise conditions. In this case the accuracy of the track information state transformation (decision) must suffer. Again, there is the problem of many other tracks present that must be also considered.

Therefore, control is performed by the higher echelons by determining which track (commodity) should be processed by which module element and how much time should be spent performing the transformation (decision) process. It should be noted that, in the SAGE system, $(m!)^n$ possible sequences of m tracks through n module elements does not exist because of precedence constraints. That is, since a track

must be processed in order through the module elements only, (m!) possible sequences exist. This is further reduced by priority constraints which will be discussed later in the report.

10. Off-line Control Subsystem:

The off-line control subsystem shown in Figure 1 performs an adaptive function for the information handling and decision processes of the center. It is shown by dotted lines to indicate that it does not perform in an on-line fashion as do the above subsystems. It is included here for purposes of completing the control loop from the system criterion back to the center and emphasizing the fact that command and control systems are learning or adaptive systems. Therefore, any mathematical modeling should be cognizant of this fact. Many of the center's information handling and decision processes are highly probabilistic. The off-line control subsystem is utilized to collect the required statistics for future augmentation of the static variables of the system and makes recommendations to the on-line control subsystem for future control of the dynamic variables.

This subsystem of SAGE must collect information on the statistics of the input material information such as weather conditions, radar inputs, input tracks, flight plans, and weapon status. It then must collect the statistics on the outcomes of the system. In SAGE, these outcomes are the number of tracks processed, tracks lost in noise, errors made on the information states of tracks, delays in track information transformation processes, and the resulting hypothetical losses incurred by these outcomes.

With these statistics, the system's functional blocks and organization may be altered for succeeding operations of the system. In addition, the operating procedures of the on-line control system would be changed. These changes would establish new track priority functions and possibly alter the allowable time delays for collecting information on these tracks. That is, the statistics collected would continually recommend new decision time delay - error trade - off functions for the human elements, when collecting information on a track.

11. Effector and Sensor Subsystems:

The effectors of the system are defined as the elements that are controlled by the output commands and decisions of the command and control center.

In the case of SAGE, these effectors are the weapons assigned to the unknown or hostile tracks.

The sensor subsystem is defined as the origin of all input material information inputs (discussed previously).

In conclusion, the functional blocks, the direct relationships, and the variables of a command and control system such as SAGE can be modeled in terms of an industrial control system. This greatly simplifies the sensitivity analysis of the information handling and transformation processes and yields insight into the on-line control of the system variables. Figure 9 shows a summary of the terminology used in this section which will be utilized in the remainder of the report.

The subsystem functional blocks were discussed in this section. In the following section the module element functional blocks will be modeled and the organizational structure and direct relationships of SAGE developed.

B. MODULE FUNCTIONS AND ORGANIZATIONAL STRUCTURE

The organizational structure of the center is defined after each of the module functions are developed. This structure defines the echelons, groups, subsystems, communication paths, information transformation sequences, and decision, control, and command authorities of the modules.

Since SAGE is already an existing system, we will redefine some of the basic modules of the system in this section. The time delay and error variables and the direct relationships between the modules are also defined. These direct relationships are then utilized in the following section for development of the indirect relationships between the functional blocks of the system.

It should also be noted that the following module functions are representative of actual SAGE functions but assumptions are made to aid the mathematical models and developments throughout the report.

1. Elements of the Modules:

Figure 10 shows a simplified module structure of the SAGE system functional blocks defined in Figure 5. Three echelons of modules are shown. Figure 11 shows a typical SAGE man-computer-display module diagram.

Functional Blocks

Computer Program Element

Display Element

Human Element

Module Element

Groups

Subsystems

Echelons

Relationships Between Blocks

Communication Links and Routes

Information Transformation Sequences

Decision, Control, and Command Authorities

Processes

Material Information Transformation

Track (Commodity) Information Transformation

Information, Decision, Control, and Command Communication

Variables

Dynamic or Static

Controllable or Uncontrollable (Noise)

Errors

Time Delays

Capacity

Sequences

Routes

Figure 9. Command and Control System Terminology

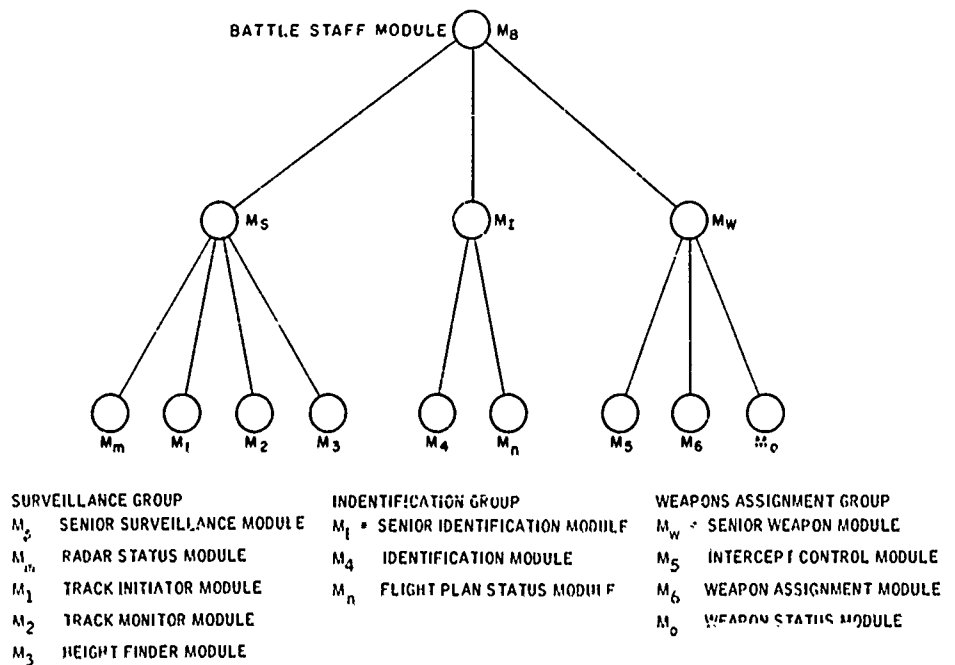


Figure 10. Simplified Module Structure of SAGE Direction Center

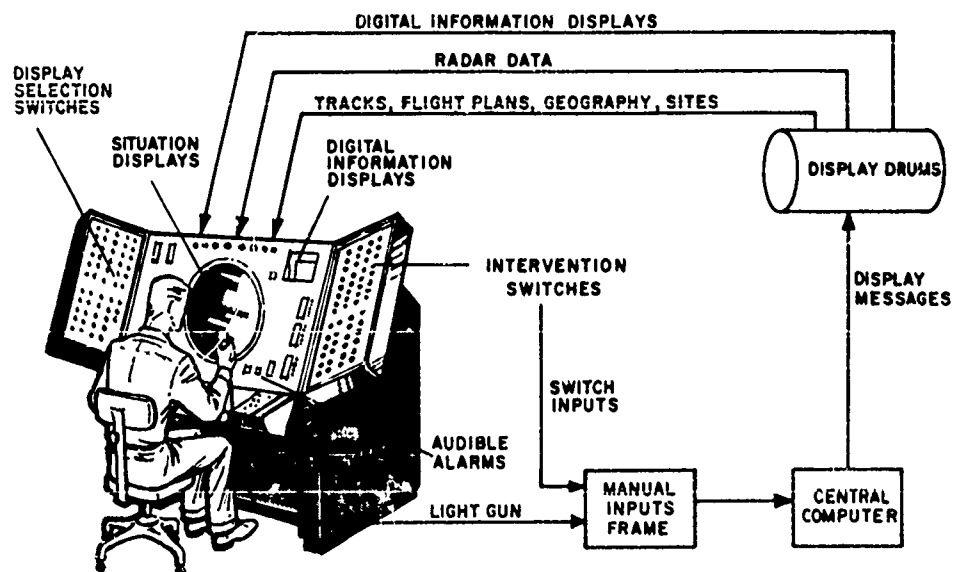


Figure 11. Man-Computer-Display Module

These modules will be defined as consisting of the three basic elements shown in Figure 12. These are:

- (1) Decision Element
- (2) Computer Element
- (3) Display Element.

The human element in Figure 11 is replaced by a decision element to eliminate any philosophical arguments throughout the report. The computer element represents the particular program in the central computer that processes the requests of the decision element. The display element represents the situation and digital displays shown in Figure 11.

There are three basic functions performed by these elements when the information state of a track is changed. Material information is processed by the computer element and sent to the display element at some rate, μ_c . The decision element then processes this information via the display element at some rate, μ_2 . This processing by the decision element results in either a decision which changes

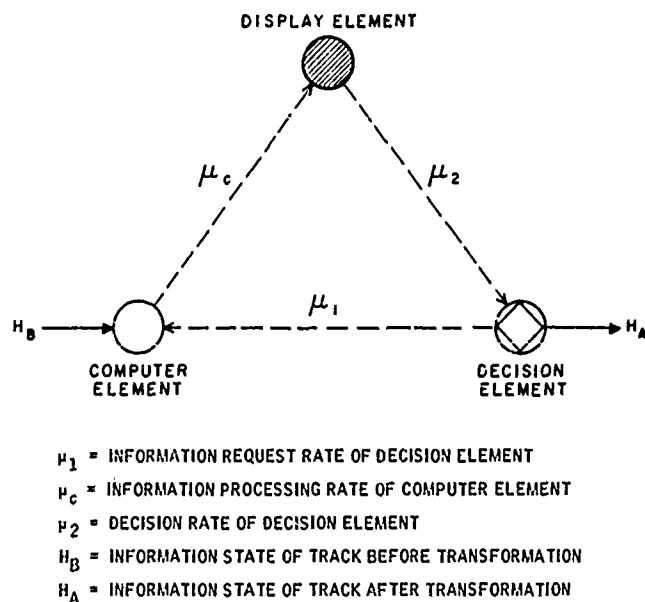


Figure 12. Basic Module Element Organization

the information state of a track from H_B to H_A or an information request. This information request is denoted by the rate μ_1 . The dynamics of these processes will be discussed in a later section.

The functions of the modules in Figure 10 will now be defined.

2. Material Information Transformation Modules:

There are three modules which transform the input material information to the SAGE Direction Center. These are the radar status module M_m , the flight plan status module M_n , and the weapons status module M_o .

These modules serve two basic functions. The first is the transformation of input material information required for the track information state transformations performed by modules M_1 , M_2 , M_3 , M_4 , M_5 and M_6 . The second is an assumed prediction function for the control modules M_s , M_I , and M_w .

a. Module M_m - Radar Status Module

The function of this module is to filter as many of the extraneous radar returns from the system as possible. It also checks the operating status of the radar sites feeding the SAGE system.

The material information transformed by this module are the radar returns. These returns can be characterized by:

- (1) True radar return from a track
- (2) Erroneous radar return from a track
- (3) Absent or delayed radar return from a track
- (4) Extraneous radar returns (clutter) from no tracks.

Therefore, the uncontrollable variables (noise characteristics) of the input material information are erroneous, absent, and/or extraneous radar returns. The information handling function performed by this module is a rejection or acceptance function.

These accepted radar returns are sent to the computer elements of modules M_1 , M_2 , and M_3 for track transformation processes. Known information on the noise levels in given geographical regions and track regions are sent to the senior surveillance module M_s for prediction and control of the track transformation processes performed by modules M_1 , M_2 , and M_3 .

b. Module M_n - Flight Plan Status Module

The functions of this module are to record, process, verify, and read into the computer all weather and flight plan information which is received external to the system.

The material information (flight plans) transformed by this module can be characterized by:

- (1) True flight plan coordinates of a friendly track
- (2) Erroneous flight plan coordinates filed on a friendly track
- (3) Absent flight plans (flight plans not filed but track flown)
- (4) Delayed flight plans
- (5) Extraneous flight plans (flight plans filed but track not flown).

The uncontrollable variables of the input material information to this module are erroneous, absent, delayed, and extraneous flight plans. The information handling functions of this module involve gathering and verify functions.

This material information (flight plans and weather information) is sent to the computer element of module M_4 for track transformation processes. This information is also sent to the senior identification module M_1 for prediction and control of the track transformation processes performed by module M_4 .

c. Module M_o - Weapons Status Module

The function of this module is to collect all information on the operational status of the weapons (effectors). The material information collected by this module can be characterized by:

- (1) Number of weapons available
- (2) Operational status of weapons
- (3) Coordinates of weapons.

Therefore, the noise conditions on the material information transformed by this module are characterized by delayed information, inoperative status, or ineffective attack coordinates. The information handling functions involve gathering and verify functions.

This material information is sent to the computer element of module M_6 for weapon assignment functions and to module M_5 for weapon control functions.

The preceding paragraphs gave the noise variables of the material information entering the SAGE system and the functions of the material information modules. Figure 13 shows a summary of these noise variables. It should be noted that even though the material information are dissimilar, their noise variables and effects on the track information transformations are very similar. One can also see the extremely probabilistic characteristics of these noise variables. As would be expected, this makes any modeling of the following track transformation modules probabilistic.

INPUT MATERIAL INFORMATION		NOISE CHARACTERISTICS		
RADAR RETURNS	ABSENT η_{11}	DELAYED η_{12}	ERRONEOUS η_{13}	EXTRANEIOUS η_{14}
FLIGHT PLANS	ABSENT η_{21}	DELAYED η_{22}	ERRONEOUS η_{23}	EXTRANEIOUS η_{24}
WEAPON STATUS	INOPERATIVE OR DESTROYED η_{31}	DELAYED STATUS INFORMATION η_{32}	ERRONEOUS STATUS INFORMATION η_{33}	INEFFECTIVE ATTACK COORDINATES η_{34}

η_{ij} = LEVEL OF TYPE j NOISE ON MATERIAL INFORMATION i

Figure 13. Material Information Noise Variables

3. Track Information Transformation Modules:

There are six modules that are defined to change the information state of a track. These are the track initiator module M_1 , the track monitor module M_2 , the height finding module M_3 , the identification module M_4 , the intercept control module M_5 , and the weapons assignment module M_6 . These modules perform the track (commodity) information state transformations, utilizing the material information from the material modules. This material information is utilized to transform the information state of a track from some initial state H_B to a succeeding state H_A (Figure 12).

a. Module M_1 - Track Initiator Module

The functions of this module are the detection of tracks and determination of their position coordinates. Module M_1 processes the radar returns accepted by module M_m and determines which returns are from actual tracks. Figure 14 shows a track history of six returns from a track and the extraneous returns

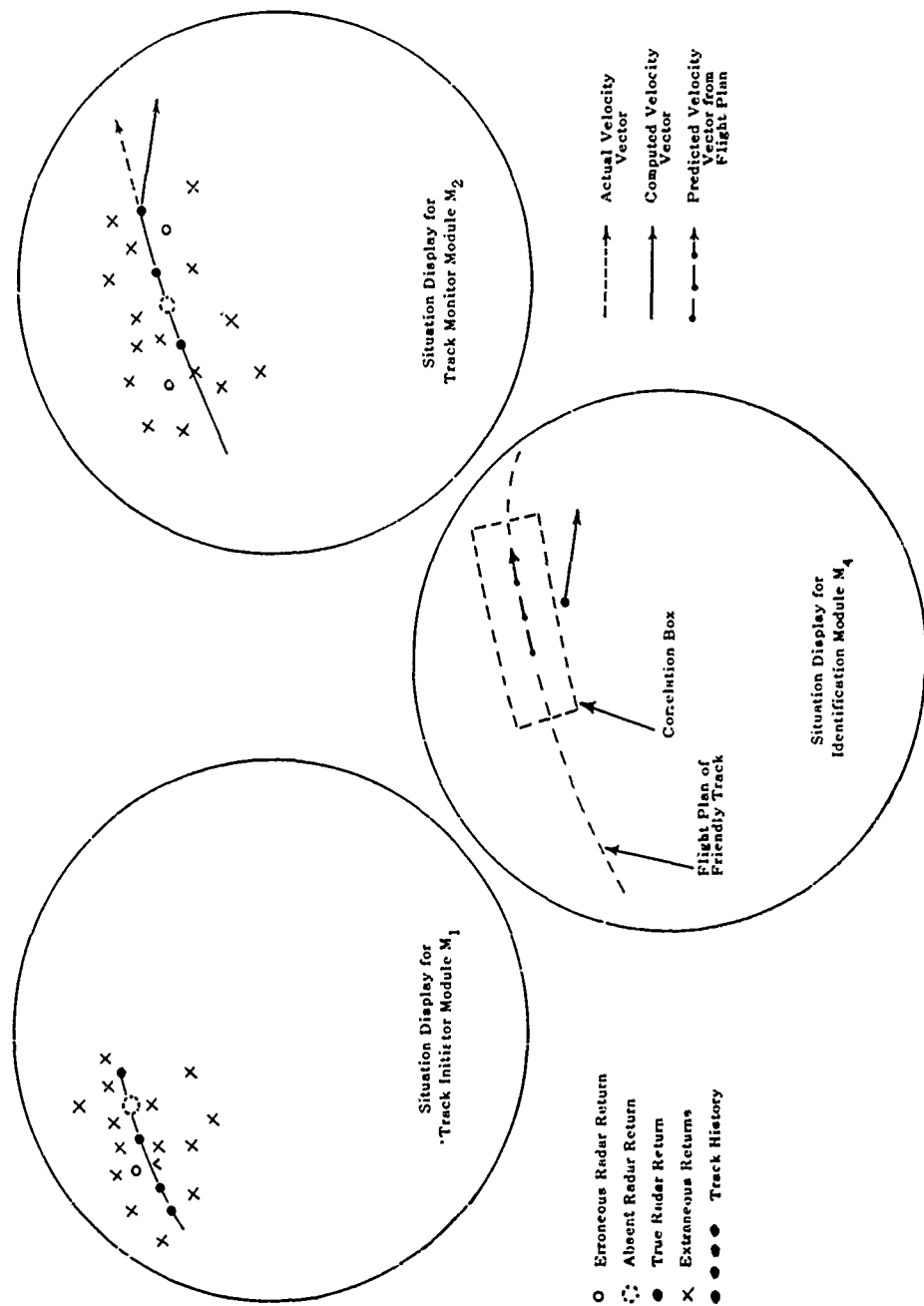


Figure 14. Situation Displays for Modules

that may appear on module M_1 's situation display (Figure 11) in a given region of a track.

Module M_1 transforms the information state of a track from a "no track" state to a "position" state. The error and time delay of this transformation process are functions of the existing noise conditions and conditional probability distribution of the decision element.

Let,

$P(x_0)$ = A priori probability that no track (x_0) is present in a given region of the display.

$P(x_1)$ = A priori probability that a track (x_1) is present in the given region of the display. $P(x_0) = 1 - P(x_1)$

y = (y_0, y_1) - Represent the two possible no track (y_0) and track (y_1) decisions of the decision element.

$P(y|x)$ = Conditional probability distribution of the decision element. This is the conditional probability that when ($x = x_0, x_1$) is the cause, that the decision element will decide ($y = y_0, y_1$) after spending T_1 time processing the returns in the given region of the display in given noise conditions η_1 .

η_1 = $f_1(\eta_{11}, \eta_{12}, \eta_{13}, \eta_{14})$ - The noise conditions existing on the material information (radar returns) utilized for the decision process.

It is assumed that the above conditional probability of the decision element is a function of the noise level η_1 and the processing time T_1 of the decision element.

$$P(y|z) = P(y|x, \eta_1, T_1) \quad (1)$$

It is assumed that the decision error is inversely proportional to the processing time T_1 and proportional to the noise level η_1 . These relationships can only be determined by experimentation.

The posteriori probabilities relate the error of the decision process and thus give the probable information state of a track after the transformation process (decision) performed by this module M_1 .

These posteriori probabilities can be developed from a theorem due to Bayes where,

$$P(x|y) = \frac{P(y|x) P(x)}{P(y)} \quad (2)$$

where,

$$P(y_i) = P(y_i|x_0) P(x_0) + P(y_i|x_1) P(x_1) \quad j = 1, 2 \quad (3)$$

The errors of this transformation process become,

$$c_{10} = P(x_0|y_1), \quad c_{11} = P(x_1|y_0) \quad (4)$$

The error c_{10} is the probability that the decision element will make a track decision when no track is present. This is the probability that the processing time T_1 was wasted in a region of the display.

The error c_{11} is the probability that the decision element will make the no track decision when a track is present. This is the probability that a track will not be detected and lost in noise.

These errors relate the sensitivity of the

- (1) Noise level η_1
- (2) Processing time T_1
- (3) Conditional probability distributions $P(y|x)$ and
- (4) A priori probabilities $P(x)$

of module M_1 's transformation process to the minimum error and minimum time goals of the surveillance node of operation. These goals were discussed in a previous section.

Given knowledge of the above factors, the error probability of module M_1 could be predicted and controlled. Control would be in the form of determining the time-error trade-off requirement of the surveillance group. This control would be carried out by the senior surveillance module M_s . The unfortunate problem in determining such a trade-off for one track is that this trade-off is also a function of other tracks present. This control will be discussed in a later section.

When module M_1 completes its transformation process it sends the track to modules M_2 and M_3 .

b. Module M_2 - Track Monitor Module

The function of this module is to assist (monitor) the automatic tracking program of the computer in the tracking function through correction of tracking difficulties in high noise conditions (Figure 14). This can be considered as a "smoothing" function.

Module M_2 transforms the information state of a track, sent from module M_1 , from the "position state" to a "velocity state." The velocity state is defined to include the position, velocity, and direction coordinates of the track. This transformation process also utilizes the material information (radar returns) accepted by module M_m .

The velocity state can be defined as being in two states. These are defined as the "acceptable" state (x_1) and the "inacceptable" state (x_0). The errors and time delay of module M_2 can then be described in a similar fashion as the track initiator model M_1 .

Let,

$$P(y|x) = P(\bar{y}|x, \eta_2, T_2). \quad (5)$$

This is defined as the conditional probability distribution of the decision element of module M_2 . This gives the time delay and probable decision y when x is the cause. In the case of module M_2 , y represents either the acceptable decision (y_1) or the unacceptable decision (y_0) under noise conditions η_2 where,

$$\eta_2 = f_2(\eta_{11}, \eta_{12}, \eta_{13}, \eta_{14}). \quad (6)$$

The cause is an acceptable velocity computation (x_1) or an unacceptable computation (x_0) from the computer element of module M_2 .

The error of module M_2 's transformation process becomes,

$$\epsilon_{20} = P(x_0|y_1) = \frac{P(y_1|x_0) P(x_0)}{P(y_1)}. \quad (7)$$

This is the probability that module M_2 will make an acceptable decision and pass an unacceptable velocity state to the remainder of the system after processing the track for time T_2 .

This error relates the sensitivity of the noise level η_2 , processing time T_2 , conditional probability distribution $P(y_1|x_0)$ and a priori probability $P(x_0)$ of module M_2 's transformation process to the time and error goals of the surveillance node of operation.

Again, control of this module's transformation process would require knowledge of the above factors and the condition of other tracks in the system.

When the decision element of module M_2 decides that the information state of the track is accurate enough for the rest of the system, it then sends

this velocity state to the identification and weapon modules M_4 , M_5 , and M_6 . When noise conditions are high, module M_2 must continue the monitoring function until the track is identified friendly. At this time it can stop monitoring and switch to another track.

c. Module M_3 - Height Finder Module

The function of this module is to determine the height coordinate of a track after module M_1 determines its position. Module M_3 performs this function by requesting the particular radar site to obtain height information via the height finding radar in a given processing time T_3 .

This module transforms the information state of a track from a position state to a "height" state. The height state is defined to include the position and height coordinates of a track (not necessarily velocity and direction).

The error ϵ_3 of this transformation is assumed dependent upon the noise conditions η_3 and the given processing time T_3 .

d. Module M_4 - Identification Module

The function of this module is to determine whether the track from the surveillance group is a friendly or hostile track.

This module transforms the information state of a track from a "track state" to an "identity state." The track state contains the position, velocity, direction, and height coordinates of a track. The identity state is defined to be either of two states. These are the hostile state (x_0) and the friendly state (x_1). This transformation process utilizes the material information (flight plans) from module M_n .

Flight plans are associated with "correlation boxes" which are displayed to the decision element upon request (Figure 14). A correlation box follows the route predicted by the flight plan. In order for a track to be correlated with a flight plan and be transformed to the friendly state, it must fall within the limits of the correlation box.

The time and error dependence of the identification transformation process can be explained by considering the noise conditions η_{24} (extraneous flight plans) or η_{21} (absent flight plans). If a number of flight plans exist in the region of

a track, correct identification of the track may require module M_4 to attempt to correlate each of the flight plans with the track. If no flight plans (absent) exist in the region of the track, module M_4 may request module M_n to check various airbases for late or changed flight plans. Each of these cases requires some time T_4 to improve the decision accuracy. It should also be noted that the identification accuracy is also dependent upon the error ($\Delta \vec{e} = \vec{R} - \vec{r}$) of the surveillance node. \vec{R} and \vec{r} represent the true and computed track coordinates respectively.

Let,

$P(x_0)$ = A priori probability that the track is hostile.

$P(x_1)$ = $1 - P(x_0)$ = A priori probability that the track is friendly.

η_4 = $f_4(\eta_{21}, \eta_{22}, \eta_{23}, \eta_{24})$ = The existing noise conditions on the material information (flight plans) to module M_4 . $\Delta \vec{e}$ is assumed zero in this discussion, but will be considered later.

$P(y/x)$ = $P(y/x, \eta_4, T_4)$ = Conditional probability of decision element of module M_4 . This is the probability that when the cause is x that the decision element will decide y after spending time T_4 gathering information in noise conditions η_4 .

The errors of this transformation process become,

$$\epsilon_{40} = P(x_0/y_1), \quad \epsilon_{41} = P(x_1/y_0)$$

The error ϵ_{40} is the probability that a hostile track is identified friendly. The error ϵ_{41} is the probability that a friendly track is identified hostile.

Therefore, the error of the identification node of operation is a function of the decision element's, conditional probability $P(y/x)$ distributions, a priori probabilities, $P(x)$, noise conditions η_4 , and the processing time T_4 . It also is dependent upon the surveillance error $\Delta \vec{e}$.

When a track is identified hostile it is sent to the weapons modules M_5 and M_6 . If friendly, module M_2 is notified and the monitoring function is assumed to cease.

e. Module M_5 - Intercept Control Module

The function of this module is to determine the intercept coordinates and weapons demand to destroy a hostile track. It then performs any weapon control functions necessary to achieve the intercept. When module M_5 determines the weapons demand and intercept coordinates, it then is assumed to request module M_6 to

perform the assignment function. If the assignment function permits module M_5 's demand, then weapon control and intercept takes place.

Computation of the intercept coordinates for various weapons are requested of the computer element until the optimum weapon-target intercept combination has been selected. This trial computation and selection process is assumed to take a processing time T_5 . The error ϵ_5 of this process is defined as a non-optimum weapon-target coordinate selection. It is assumed that this error is a function of the noise conditions $\eta_5 = f_5(\eta_{31}, \eta_{32}, \eta_{33}, \eta_{34})$ and the processing time T_5 . That is, the more trial intercept computations tried, the less probable will be a non-optimum selection (or error ϵ_5).

The final intercept control effectiveness is then some function of this error ϵ_5 and the surveillance node error Δe on the target track.

f. Module M_6 - Weapons Assignment Module

The function of this module is to determine the proper assignment of weapons to hostile tracks. These assignments are then communicated to module M_5 for intercept control.

The assignment function can be thought of as changing the information state of a track from the "hostile" state to the "weapon" state via module M_5 and M_6 .

The assignment effectiveness depends upon the noise conditions $\eta_6 = f_6(\eta_{31}, \eta_{32}, \eta_{33}, \eta_{34})$ on the material information from module M_0 . This effectiveness can be related to an error of assignment in a similar manner as the errors of the previous modules. This error is also a function of a processing time T_6 . However, now this processing time is a computational process involving the assignment or transportation problem of linear programming.

Let,

- a_i = number of weapons available at location i (supply)
- b_j = number of weapons required at hostile track destination j (demand)
- z_{ij} = number of weapons assigned from location i to hostile destination j
- c_{ij} = cost of assigning one weapon from location i to destination j .

Then, if the costs of assignment are linear functions, the assignment problem becomes one of minimizing the total assignment cost C_A subject to the

supply and demand constraints. That is, module M_w seeks to minimize,

$$C_A = \sum_{i,j} c_{ij} z_{ij}$$

subject to the constraints,

$$\begin{aligned} \sum_j z_{ij} &\leq a_i & i = 1, 2, \dots, m \text{ locations} \\ \sum_i z_{ij} &\leq b_j & j = 1, 2, \dots, n \text{ destinations.} \end{aligned}$$

For example, consider the problem where two hostile tracks ($n=2$) exist and two weapon locations ($m=2$) have one weapon at each location. In this case, there exist four weapons assignment possibilities of which one may minimize the total cost C_A .

If module M_w takes the time to perform the problem iterations involved in the above assignment problem, the probability of an assignment error is zero. However, such iterations take a certain processing time T_6 depending upon the number of hostiles and weapons available and their operating status. Therefore, the problem now arises to how much time T_6 module M_6 should take to perform the assignment function. That is, an optimum time delay - assignment error trade-off also exists with this module in the same manner as with the previous modules. This assignment error will be defined as ϵ_6 , and will be assumed to be a function of the processing time T_6 and noise conditions η_6 .

Therefore, the effectiveness of the weapons assignment node of operation is assumed to be a function of module M_5 's error ϵ_5 , module M_6 's error ϵ_6 and the surveillance node error Δe . This combined error will be defined as $\epsilon_w = f_w(\epsilon_5, \epsilon_6, \Delta e)$.

g. Time - Error Relationships of Transformations

From the preceding definitions of the information transformation modules, it can be seen that the minimum time and minimum error goals of the surveillance, identification, and weapons assignment nodes are dependent upon the noise conditions existing on the material information (radar returns, flight plans, weapons status) when the track information state transformations take place. If decisions are made early (minimum time goal), before enough information has been acquired and processed, errors may result and the minimum error goal suffers. If decisions are delayed to acquire and process more information to minimize errors, the time goal suffers.

Therefore, the noise conditions create the functional relationships between the time and error goals of each node of operation of the system.

In addition, these same noise conditions create cross-coupling effects between module operations. This will be discussed in the following section when the indirect relationships are developed.

It should also be noted that each track is assumed to be passed to succeeding modules in a discrete manner after its information state has been changed. This treats the tracks as discrete commodities as stated previously and eases the analysis of information flow in the system.

4. Control Modules:

There are four control modules. These are the senior surveillance module M_S , the senior identification module M_I , the senior weapons module M_W , and the battle staff module M_B .

Control can take the following forms in the SAGE model:

- (1) Track Control
- (2) Transformation time control
- (3) Transformation sequence control.

Track control consists of the control modules determining which track the transformation modules should process out of a number of existing tracks.

Time control consists in determining when the transformation modules should make a decision on the information state of a track or when a decision should be accepted from a module.

Transformation sequence control consists of the control modules determining how n tracks should be processed by the six transformation modules M_1 , M_2 , M_3 , M_4 , M_5 , and M_6 .

Track and time control is discussed in the sensitivity analysis section of the report. Sequence control is discussed in the last section of the report.

In order for the control modules to perform the above control possibilities, the input noise conditions on the material information must be known. Knowledge of a priori probabilities of track inputs and the conditional probabilities of the transformation module decision elements increases such control effectiveness

but is not a limiting factor. A priori knowledge of tracks is obtained from flight plan status module M_n .

Control is also possible when the noise conditions are known and decision delays under these conditions are known or specified. This will be discussed in the last section of the report.

5. System Organization:

With the previous definitions of the module functional blocks the system organizational structure can be developed. This structure should define:

- (1) Communication paths between modules
- (2) Command, control, and decision authorities and paths
- (3) Information transformation sequences.

It should be noted that the organizational structure is developed from knowledge of the above direct relationships between the modules. The indirect relationships are developed after this structure is developed. Figure 15 shows the basic module organization.

The control paths are shown from the control modules M_B , M_S , M_I , and M_W to the transformation modules M_1 , M_2 , M_3 , M_4 , M_5 and M_6 . The track information state sequence is shown through the transformation modules. Material information required for these transformations is shown from the material modules M_m , M_n , and M_o . Prediction information giving the noise conditions on the material information is shown from the material modules to the control modules.

Finally, two other assumed communication paths are shown. These are the intergroup goal communication paths and the blocking communication paths. The intergroup goal communication is assumed to keep the priority and time constraints of each group compatible with the overall system criterion. That is, as the weapon status and flight plan status change, these conditions may require track priorities in various regions of the display to change. The blocking communication is assumed to communicate the condition of a track or tracks back to preceding modules. For example, in high radar noise conditions module M_2 will not send a track to module M_4 until the track information state is accurate. This means module M_1 has to perform a monitoring function on new input tracks until M_2 is free. When M_2 is free, it is assumed to communicate to M_1 and take the new track. This is defined as blocking communications. This is discussed more thoroughly in the next section.

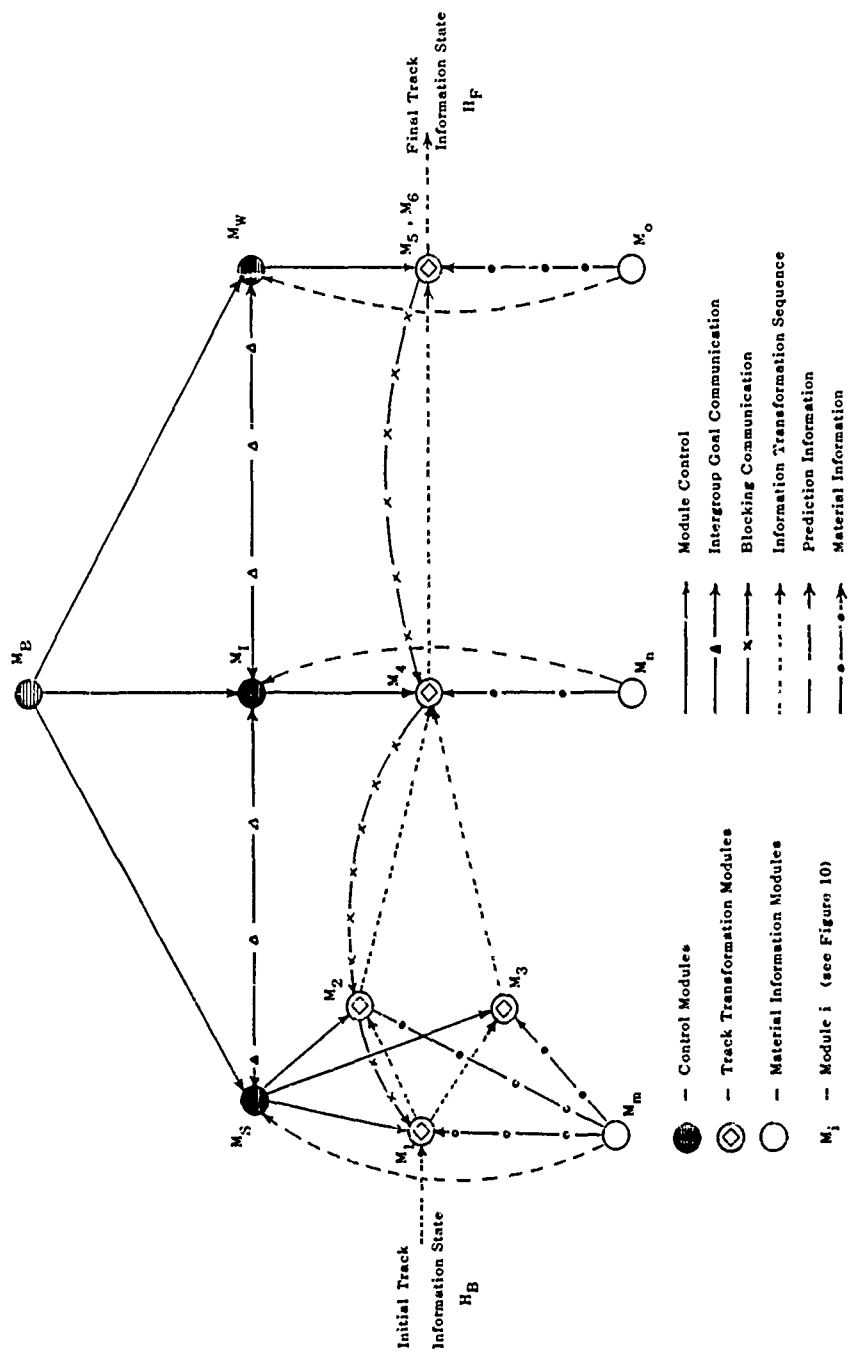


Figure 15. SAGE System Module Organization Structure

VII. SYSTEM VARIABLES, GOALS, AND INDIRECT RELATIONSHIPS

The organizational structure developed in the last section gave the direct relationships between the modules. It now remains to discover any indirect relationships which might exist between the modules. This is necessary before the sensitivity of any module's decision error or delay can be related to the system criterion.

For example, the weapons modules directly affect any interception errors ϵ_w . However, the time delay of modules M_1 , M_2 , or M_3 also could have contributed to this error indirectly. The goals of each of the modules may also indirectly affect the criterion. Therefore, the establishment of such goals must be cognizant of how such goals will affect the modules and finally the system criterion.

In this section, the concept of digraphs and matrix techniques will first be reviewed for finding indirect relationships and cycles of relationships in the module organizational structure. Then the minimum time and error goals of the modules will be defined. Finally, the digraphs showing some indirect relationships between the goals, modules, and the system criterion will be shown. These indirect relationships will more clearly define the blocking and goal communication paths shown in Figure 15.

A. ACYCLIC DIGRAPHS

Once the direct relationships between the goals (constraints) and modules and between the modules have been developed from the organizational structure, the indirect relationships can sometimes be determined by directed graphs (digraphs) and matrix techniques.

For example, consider the digraph discussed in the system analysis section and presented in Figure 16a.

Four elements of an organization are shown as nodes e_1 , e_2 , e_3 , and e_4 . For the moment, these elements may represent a computer element, human element, module element, or a variable of one of these elements. The goal for element e_3 is also shown as a node g_3 - this goal may represent a time or error constraint of element e_3 .

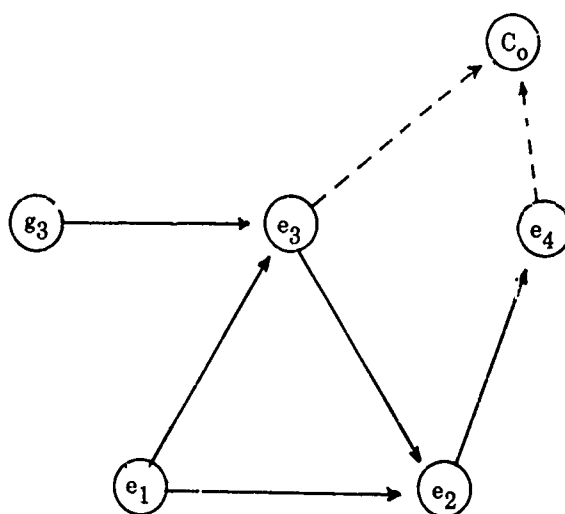


Figure 16a. Acyclic Digraph

The criterion is some direct function of the elements e_3 and e_4 and is shown as node C_0 . The vectors shown between the nodes represent direct relationships between the nodes. If node i directly affects node j in any manner, a vector is drawn between the nodes.

A connective or first order direct relationship matrix C can now be defined for this digraph, where the (i, j) entry of the matrix is one if node i directly affects node j in any manner.

	C_0	g_3	e_1	e_2	e_3	e_4
C_0	0	0	0	0	0	0
g_3	0	0	0	0	1	0
e_1	0	0	0	1	1	0
e_2	0	0	0	0	0	1
e_3	1	0	0	1	0	0
e_4	1	0	0	0	0	0

All of the n th order indirect relationships can now be detected by performing a logical or boolean matrix product of this first order matrix.

The boolean product $A \wedge B$ of the boolean matrices A and B is that boolean matrix whose (i, j) entry is

$$\vee_k (a_{ik} \wedge b_{kj})$$

where \vee and \wedge in this expression denote the boolean operations of max and min respectively. Boolean sums and products of boolean matrices are formed in the same manner as ordinary matrix sums and products except that $+$ is replaced by \vee and \times by \wedge . (5)

Therefore, the n th order indirect relationships of the above graph are given by

$$C_n = C_{n-1} \wedge C \quad (12)$$

where the boolean matrix multiplication is denoted by \wedge . For example, the second order ($n=2$) indirect relationships become,

$$C_2 = C \wedge C = \begin{matrix} & \begin{matrix} C_0 & g_3 & e_1 & e_2 & e_3 & e_4 \end{matrix} \\ \begin{matrix} C_0 \\ g_3 \\ e_1 \\ e_2 \\ e_3 \\ e_4 \end{matrix} & \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{matrix}$$

Thus, from C_2 we see that

- g_3 affects C_0 via its effect on element e_3
- e_1 affects C_0 via its effect on element e_3
- e_1 affects e_2 via its effect on element e_3
- e_1 affects e_4 via its effect on element e_2
- e_2 affects C_0 via its effect on element e_4
- e_3 affects e_4 via its effect on element e_2 .

Third and fourth order relationships can be found in a similar fashion of matrix multiplication. There exist no fifth order relationships and the only fourth order relationships are $g_3 \rightarrow e_3 \rightarrow e_2 \rightarrow e_4 \rightarrow C_0$ and $e_1 \rightarrow e_3 \rightarrow e_2 \rightarrow e_4 \rightarrow C_0$. The matrix multiplication is carried out until all elements of the matrix are zero. This then gives all the indirect relationships. In the case above, this would be the fifth order matrix $C_5 = O$.

These relationships are readily seen from the graph. However, for command and control systems, such as SAGE, the detection of these indirect relationships

is not always possible from visual investigation of the direct relationship graphs. In this case, such matrix operations can be performed by hand or simulation.

It should be noted that the digraph shown contains no cycles of relationships and is therefore defined as an acyclic digraph. In the case of acyclic digraphs, the C_n matrix will eventually go to zero. However, when cycles exist in the indirect relationships, other means must be utilized to first detect such cycles and then reduce the digraph for analysis.

B. CYCLIC DIGRAPHS

Frequently, in command and control systems, cycles of indirect relationships exist in the system's digraph. Detection of such cycles is important for two reasons. First, if the cycle is isolated from the rest of the system, its goals can be established independently of the other elements and goals of the system. Also, the sensitivity analysis, relating this cycle to the overall system criterion, can be carried out independently. Second, if cycles are connected or related to the other elements of the system, these cycles and the elements of the cycle cannot be treated independently from the rest of the system when developing the sensitivity analysis.

We now show a method of detecting single cycles by considering the simple cyclic digraph in Figure 16b.

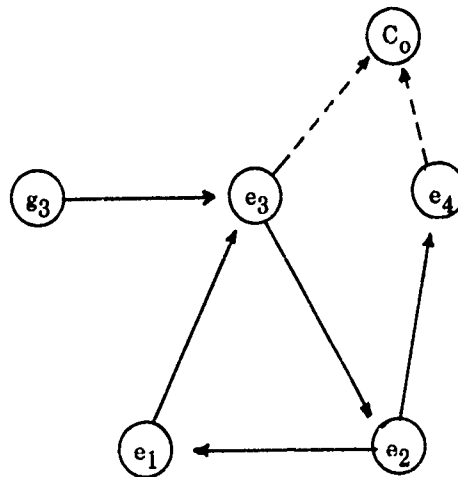


Figure 16b. Cyclic Digraph

The corrective or first order matrix for this digraph becomes:

	C_0	g_3	e_1	e_2	e_3	e_4
C_0	0	0	0	0	0	0
g_3	0	0	0	0	1	0
e_1	0	0	0	0	1	0
$C = e_2$	0	0	1	0	0	1
e_3	1	0	0	1	0	0
e_4	1	0	0	0	0	0

A procedure for finding the cycle (or residual matrix) of the above digraph is given in reference (8) and demonstrated here. The steps of the procedure are:

- (1) Check every row in the matrix C whose entries are all zero. If the i th row is a zero row, delete this i th row and the corresponding i th column from the C matrix.
- (2) Check every column in the matrix C whose entries are all zero. If the j th column is a zero column, delete this j th column and the corresponding j th row from the C matrix.
- (3) Continue the above two steps until no zero columns or rows are left. The residual matrix will contain the cycle of the digraph.

For example, in the given C matrix the first row is the zero row. This row and the corresponding first column are deleted, giving:

	g_3	e_1	e_2	e_3	e_4
g_3	0	0	0	1	0
e_1	0	0	0	1	0
e_2	0	1	0	0	1
e_3	0	0	1	0	0
e_4	0	0	0	0	0

Next we see that the first column and last row are a zero column and row. The first row and column and the last row and column are now deleted, giving:

	e_1	e_2	e_3
e_1	0	0	1
e_2	1	0	0
e_3	0	1	0

This is the residual matrix of the digraph which represents the cycle $e_1 \longrightarrow e_3 \longrightarrow e_2 \longrightarrow e_1$.

The above shows a simple example of one cycle of relationships existing in the system's digraph. As will be shown, the SAGE module digraph can be one which contains a number of such cycles or closed loops, two or more "tangent" closed loops with common nodes, and various other combinations. These complicated cases still remain a problem in digraph theory and represent possible future work. Such problems have been investigated extensively by Harary. ⁽⁷⁾

C. MODULE VARIABLES AND GOALS

The variables of each module's operation were discussed in the previous section and are briefly listed below:

- (1) Track order
- (2) Track transformation times $T_1, T_2, T_3, T_4, T_5, T_6$
- (3) Track transformation errors $\epsilon_1, \epsilon_2, \epsilon_3, \epsilon_4, \epsilon_5, \epsilon_6$
- (4) Material information noise conditions $\eta_1, \eta_2, \eta_3, \eta_4, \eta_5, \eta_6$.

The free or controllable variables of the modules comprise all but the last. As can be seen, the main variables of each module's operation are the time, error, and order variables.

Achievement of the system criterion is a function of the time delays, errors, and track order of each module. In order to see the indirect effects of each module's variables on the other modules and the system criterion, we can consider two extreme goals or constraints for each module. These are a minimum time goal and a minimum error goal.

A minimum time goal is defined as constraining the processing (a decision) time on a track regardless of the noise conditions. This means a decision must be made within some processing time T_i when the module starts processing the track. The minimum time goal can be seen to favor early interception of hostile tracks and is defined as effective in this sense. However, it may also result in decision errors when noise conditions are high, since decision times are constrained. This may result in identification errors and the interception of friendly tracks and thus ineffective.

The minimum time goals for the modules have the advantage of reducing the blocking effects between modules under high input rates or noise conditions. This means that the sensitivity analysis would develop the loss rate due to errors and module delays within the system. That is, the probability of tracks being processed on a first come - first serve basis would be high and any delays due to waiting lines (queues) would be small.

A minimum error goal is defined as constraining a module's decision error within some confidence level regardless of the required processing time. That is, a decision on a track is not made until enough information has been processed to assure some level of confidence. The minimum error goal results in a low level of per track errors but would result in high loss rates resulting from hostile tracks not being processed early or not processed at all.

The minimum error goal results in blocking effects and waiting lines will form in front of the modules and system. Now the waiting line delays must be considered as to their effect on the system criterion.

Therefore, it can be seen that optimum time-error trade-offs exist for each module. In addition, these are different for different track classes. That is, each track will have a priority function associated with it that gives the order in which it should be processed with respect to other tracks and the time that should be spent on it.

These priority and time constraints change continuously with the performance statistics of the system and on-line with the decisions which are being made. We now consider the indirect effects that may exist due to such priorities and time constraints on the module operations.

D. SAGE INDIRECT RELATIONSHIPS AND DIGRAPHS

The previous sections have treated the tracks as discrete quantities (commodities) with discrete information states to make the decision sequences of the modules more clear. The indirect relationships can also be more clearly seen if this assumption is again made.

1. Single Module:

Figure 17 shows a digraph representation of these modules. We will first describe the single module M_i in general terms before preceding to the multiple module digraphs.

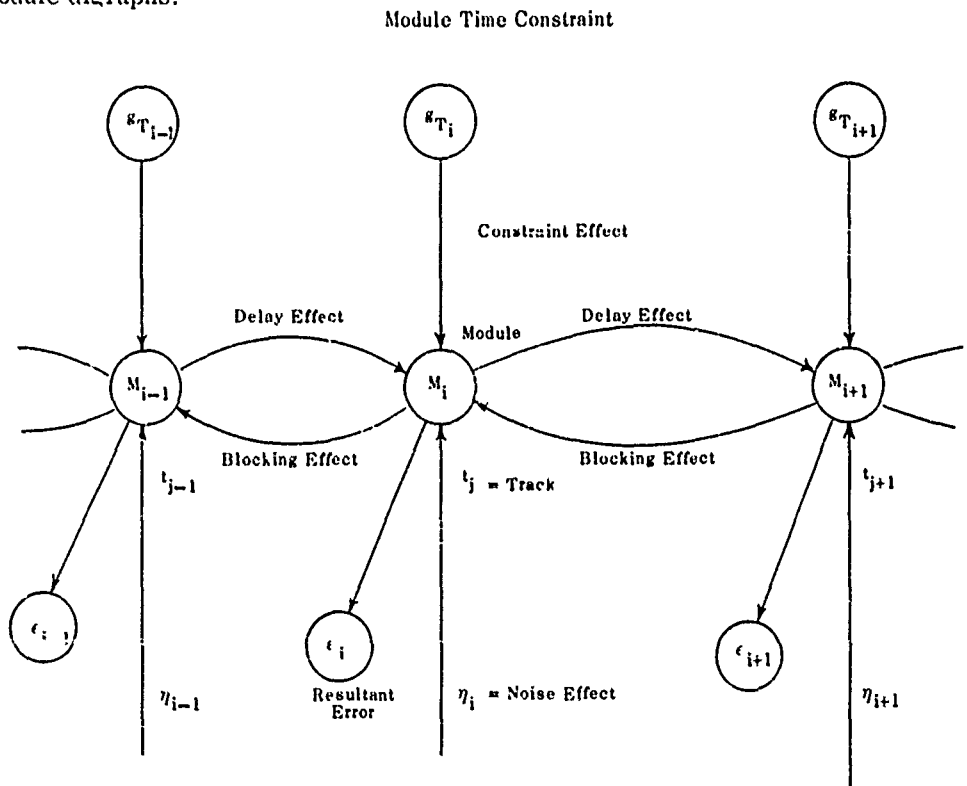


Figure 17. Module Digraph Representation of Interrelationships

Module M_i is shown processing track t_j . The time constraint for processing this track is shown as node g_{T_i} . The resultant error from processing this track is node e_i . The existing noise conditions on this track is shown as η_i . Therefore, under noise conditions η_i , module M_i will process track t_j . If noise conditions are low, it may complete processing track t_j within the g_{T_i} constraint and pass it to

module M_{i+1} with an error ϵ_i . If noise conditions are high, module M_i can take up to time g_{T_i} and then must make a decision and pass the track. The resultant error ϵ_i may be higher in this case. Therefore, the error ϵ_i can be seen to be a variable function of the noise conditions η_i and either the processing time T_i or the time constraint g_{T_i} .

2. Multiple Modules:

When each of the modules are processing a track or tracks, cross-coupling effects can effect the module performance.

Consider the $M_i \longrightarrow M_{i+1} \longrightarrow \epsilon_{i+1}$ relationship. This is an indirect effect on the error ϵ_{i+1} due to a delay. That is, if track t_j is delayed by module M_i , it may reach module M_{i+1} late. Due to the time constraint, t_j has to be processed within time $g_{T_{i+1}}$. If noise conditions η_{i+1} are high, this results in an error ϵ_{i+1} or track t_j . Thus, we define this delay effect as having a direct effect on module M_{i+1} and an indirect effect on the error ϵ_{i+1} .

Consider the $M_{i+1} \longrightarrow M_i \longrightarrow \epsilon_i$ relationship. This is defined as a blocking effect. When module M_i finishes track t_j , it will send it to module M_{i+1} . However, if module M_{i+1} is busy processing track t_{j+1} , then module M_i may not be able to pass track t_j . In this case, blocking occurs. If track t_j is still within the time constraint g_{T_j} , then such waiting is permissible. However, if the processing time limit on track t_j has been reached, then a decision has to be made for module M_{i+1} to pass track t_{j+1} and accept track t_j or for module M_i to continue processing track t_j . As can be seen, if module M_i continues processing t_j past the time constraints, it may improve the error ϵ_i if noise conditions η_i are high. Thus, the blocking effect is defined as having a direct time delay effect on module M_i and an indirect effect on the error ϵ_i of track t_j .

It can be seen that such logic can be extended for a number of modules. For example, module M_{i-1} had an indirect effect on the accuracy of module M_{i+1} when processing track t_{j+1} by its delay effect via module M_i . Also, module M_{i+1} has an indirect effect on module M_{i-1} by its blocking effect on module M_i which, in turn, effects the blocking of module M_{i-1} .

The above has described strong blocking effects. Weaker blocking effects might be defined when module M_i had to process both tracks t_j and t_{j+1} . (This might be the track monitor module M_2 in the SAGE system.) When M_{i+1} finishes processing track t_{j+1} , then module M_i may no longer have to process both tracks but can now concentrate on t_j . This is defined as a weak blocking effect when a module

has to time share its processing time between two tracks. That is, track t_j may have to wait until t_{j+1} is processed. If a time constraint exists for t_j , its accuracy is affected.

3. SAGE Digraph:

We now consider extending the general module description to the SAGE system and developing its digraph, Figure 18. The control modules M_S , M_I , and M_W and the height finding module M_3 are not shown for simplicity. The control modules can be thought of as continually changing the goals shown. This would be done continuously as the system operates and decisions are made by modules M_1 through M_6 . This requires the intergroup goal communication shown in Figure 15.

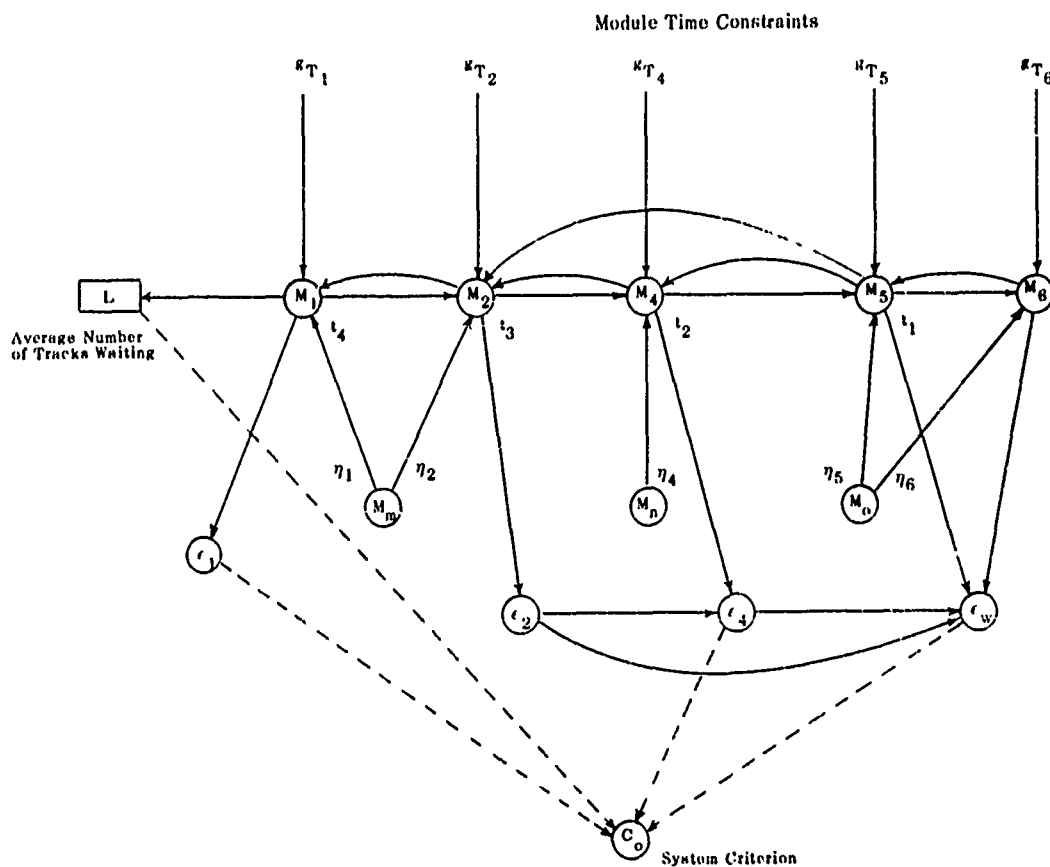


Figure 18. SAGE Digraph Showing Indirect Module Relationship

Achievement of the system criterion is shown to be directly a function of the average number of tracks waiting to be processed L , the tracks lost in noise $\epsilon_1 = \epsilon_{11}$, the identification errors $\epsilon_4 = (\epsilon_{40}, \epsilon_{41})$, and the weapons effectiveness (errors) $\epsilon_w = f_w(\epsilon_5, \epsilon_6)$.

Each of the material information modules M_m , M_n , and M_o are shown directly affecting the corresponding transformation modules. The time constraints (goals) are also shown directly affecting each of the transformation modules and indirectly affecting their errors ϵ_1 .

A track is shown being processed by each module. One track t_j is used to represent one or a number of tracks. However, to discuss the indirect effects it is much easier to talk in terms of one track. The effects of the time constraints and noise modules on the track transformations has been discussed. Therefore, only some of the less apparent effects will be briefly explained for this SAGE digraph.

a. Module M_1

M_1 affects the number L of tracks waiting to be processed. Module M_2 affects when track t_4 can be sent to module M_2 . If noise conditions η_1 are high on track t_4 , then M_1 must perform the monitoring function until track t_3 has been processed.

b. Module M_2

M_2 has a time delay effect on module M_4 and thus an indirect effect on its error ϵ_4 . It also has a direct effect on this error. This direct effect is due to module M_2 performing the coordinate determination (monitoring) for track t_2 . This direct effect is shown by the vector $\epsilon_2 \longrightarrow \epsilon_4$.

Modules M_4 and M_5 affect M_2 's processing operation by the time sharing effect previously discussed. That is, when module M_4 identifies track t_3 as friendly, it can be dropped from the system. Module M_2 can then stop the monitoring function on track t_2 and concentrate on track t_3 . Module M_5 has a similar effect. When track t_1 is identified friendly by visual contact from an interceptor, it can also be dropped from the system. Also, when intercept has been made with a hostile track t_1 , module M_2 can stop its monitoring function. Therefore, the processing time delay of module M_5 directly affects module M_2 . This direct effect is shown by $M_5 \longrightarrow M_2$. It also has an indirect blocking effect on module M_2 via module M_4 . This is shown by $M_5 \longrightarrow M_4 \longrightarrow M_2$.

It should also be noted that module M_2 has both an indirect and direct effect on the error ϵ_w . The indirect error effect is due to its time delay effect. This is denoted as $M_2 \longrightarrow M_3 \longrightarrow M_5 \longrightarrow M_6 \longrightarrow M_5 \longrightarrow \epsilon_w$. That is, track t_1 must travel this sequence before a weapon is assigned. If a delay results at M_2 , then less time is permitted for module functions M_5 and M_6 due to the time constraint g_{T_6} . This may result in an assignment or weapon selection error. The direct effect is via the monitoring function that module M_2 performs for module M_5 on track t_1 . This error Δe also affects the total effectiveness error ϵ_w . This direct effect is shown by $\epsilon_2 \longrightarrow \epsilon_w$.

c. Module M_4

Module M_4 affects when module M_2 can stop monitoring track t_2 . M_4 has a time delay effect on module M_5 and thus an indirect error effect on ϵ_w . It also has a direct error effect on ϵ_w . This is due to the probability that a friendly track may be identified hostile and destroyed. This direct effect is denoted $\epsilon_4 \longrightarrow \epsilon_w$.

Module M_5 affects M_4 by its visual identification process of unknown tracks. When track t_1 is identified by M_5 , module M_4 can cease attempting to identify track t_1 and can concentrate on track t_2 . This was earlier defined as a weak blocking effect and is denoted by $M_5 \longrightarrow M_4$.

d. Modules M_5 and M_6

Module M_5 has a time delay effect on module M_6 and the above weak blocking effect on M_4 . Module M_6 in turn has a time delay effect on M_5 . That is, intercept cannot be performed by M_5 until the assignment function by M_6 has been performed. Both of these modules effect the error ϵ_w directly.

e. Track Order Variable and Digraph Representation

The indirect relationships presented above referred to only the decision time delay and error variables of the modules. The other variable which is of concern in the digraphs is the track order variable. This requires priority functions to be established for each track in addition to the track time constraints g_{T_1} . The priority functions and time constraints change both in an off-line and an on-line real time manner. To understand this on-line changing, the system information pattern must be defined.

The information pattern of the system at any time defines the information state of each track or tracks at each module. (This is the pattern which the battle staff module M_B is presented at any time.) The information pattern also includes the radar, flight plan, and weapon status information states or noise conditions. This information pattern is a result of previous module decisions and in turn affects the succeeding time goals g_{T_1} and track priorities of each module.

For example, when a track is identified friendly by an interceptor two effects result in the system. First, modules M_2 and M_4 can cease with the monitoring and identification functions. This is defined as increasing the monitoring effectiveness for other tracks in the system, since time sharing between a less number of tracks results. It also decreases the time delay of new tracks processed by M_2 (sent from M_1) since time sharing is again decreased. This first effect is the time-error effect discussed in the preceding paragraphs.

The second effect concerns the priority and time constraints. The coordinates of the above interceptor have changed during the identification process. These new coordinates are defined to affect the priority and time constraints of module M_2 . That is, with the interceptor in one position region of the sector, its effectiveness in other regions is decreased. This then affects the priorities of tracks in these regions and the allowable processing times g_{T_1} for decisions.

The change of module M_2 's goal g_{T_2} is defined to be indirectly affected by weapons status module M_O via the control modules M_W and M_S . That is, the senior weapons module M_W collects the weapon status information (interceptor position) from M_O and communicates this information to control modules M_S and M_I which in turn change the priority and time constraints or goals g_{T_1} for their transformation modules M_1 , M_2 , and M_4 . This communication is defined as the inter-group goal communication path shown in Figure 15.

If a track is identified unknown or hostile at any time by module M_4 it can be seen that this decision immediately affects the operation of modules M_5 and M_6 . This decision also affects the priority rule and time constraints of modules M_1 and M_2 . That is, based upon a priori probabilities of enemy raid patterns, tracks in regions of the display may change in priority. This then increases the processing time in some display regions and decreases (blocking effect) the processing of tracks in other regions.

Therefore, as decisions are being made by the modules, these decisions may be affecting the succeeding time constraints and priorities of the other module track decisions. To simplify the digraph representation such relationships were not shown.

4. Conclusion:

The digraph for the SAGE system demonstrates how the indirect relationships existing between the decision makers might be modeled and discovered from known direct relationships. The industrial control system analogies permit the tracks to be handled discretely in digraph formulations. This aids the detection of indirect error and delay relationships.

Further work is required in digraph and matrix techniques to systematically detect the cycles of relationships in the system. Also, how to model the effects of changing information patterns in digraph form for a number of different tracks of different priorities still requires further study.

The sensitivities of priority function and constraint changes to the system criterion must be known before such changes can be correctly made for succeeding system operations. The sensitivities must include all the indirect effects on the other module variables of the system. This is because one module's operation may not only affect the system criterion directly, but, as shown, may also affect it indirectly via its effect on other module operations. This is why such indirect relationship analysis is important.

When a module's sensitivity to the system criterion is known, its constraint or goal g_{T_i} and priority functions, for different track classes, can be changed correctly. This change decreases the overall loss of the system the next time the system operates under similar track situations (or information patterns).

The sensitivity of the module operations to the system criterion is developed in the next section, where the errors ϵ_1 , ϵ_4 , and ϵ_w and the average number of tracks waiting L are related to the system criterion loss functions.

VIII. SYSTEM SENSITIVITY ANALYSIS

The sensitivity analysis is the next step in the system analysis procedure. This analysis relates the effects of the module goals or constraints and the module variables to the overall system criterion. These variables are track processing times, errors, and track processing order. They exist in each node of operation. Therefore, similar problems exist in each node of operation of the system.

The decision elements of the surveillance modules must make accurate decisions in time intervals not so long that the information state of a track will eventually be received by the weapon assignment modules too late for effective use. The senior surveillance module M_s must allocate the times spent on each possible track, and the transformation modules M_1 , M_2 , M_3 must devote these times according to some optimum ordering.

The same problems are faced by the identification modules M_1 and M_4 .

The weapons assignment module M_6 must allocate weapon-resources for hostile tracks and therefore is cognizant of rules of ordering. Finally, the intercept control module M_5 must also be cognizant of rules of selection when determining the weapon demands.

Thus, it is seen that the three nodes of operation present similar problems of optimum procedure. It was also seen from the last section that these nodes of operation are cross-coupled. This makes sensitivity analysis difficult.

The ultimate purpose of the sensitivity analysis when the variables have been defined is to permit the establishment of optimum procedures within the nodes of operation. Since such modeling must be mathematical in character, it is evident that optimization must be carried out with respect to a quantitative loss criterion. When all figures of merit or loss have been defined for each node of operation, the sensitivity modeling is complete from a mathematical standpoint.

The first section of the sensitivity analysis considers the loss functions for the module decision errors under the various noise conditions. A strong minimum or constant time constraint is utilized so that delay and blocking effects can be neglected.

The second section then discusses these delays and blocking effects by considering a strong minimum error constraint for the decision processes.

A. SENSITIVITY ANALYSIS OF NODES OF OPERATION

The loss functions must be established for each node of operation and must be cognizant of the interrelationship between the nodes. If time constraints are established which permit neglecting the blocking effects, the sensitivity analysis can be performed by treating only the direct relationships between the nodes of operation.

When the direct relationships of the system exist only in a forward sense, it means that any mathematical modeling for the purpose of sensitivity analysis should begin with the interception and weapons assignment node and be carried backwards through the system to the surveillance node.

The conventional approach of considering SAGE module functions in the order by which a given track is acted upon is not satisfactory for the purposes of establishing loss functions and sensitivity analysis.

The interception node will be the starting point of the sensitivity analysis and will be regarded as an integral part of the system, for it is the objective of the system.

1. Weapons Assignment and Interception Nodes of Operation:

The criterion of the SAGE system was defined as the protection of one city and the prevention of erroneous intercepts with friendly aircraft. The interception resources for this criterion consist of two tactical interceptors.

Since the overall criterion of the system is to protect the one city, it is considered that there is a loss of unity when the enemy carries out a successful attack on the city. We also assume that the enemy may do this an unlimited number of times, i.e., the city can be "destroyed" many times. This is in recognition of the fact that, in reality, there are many important targets which the enemy might attack.

There is also a loss associated with the unintentional destruction of a friendly track, but this loss is difficult to fix quantitatively because its relationship to the defense of the city is complex and not singular and possibly even nonexistent.

For example, there is little apparent loss, from the point of view of war values, associated with the destruction of a commercial airliner. It would ap-

pear that such a loss could be equated with the loss of a miniscule portion of the city, for example, an apartment building. On the other hand, there are other considerations, having little to do with conventional war values, which would suggest that greater importance should be associated with such an error. This extreme is relaxed by assuming wartime conditions, so miniscule weight will be attached to such an error for the purposes of the present discussion.

It is even more difficult to attach a quantitative loss to the destruction of friendly military craft when such craft are not implicated in the defensive arm. When the craft are part of the offensive capacity, the loss must be based on the complex relationships between the offensive and defensive phases of strategy.

These considerations are included here to make clear the basis for the quantitative values which modeling and analysis of SAGE depend on.

For the purposes of this model, the possibility of destroying commercial aircraft will be disregarded, and a loss α will be associated with the destruction of any friendly military craft not engaged in defense.

In order to further develop the interception model, we shall assume that the "attack" command is given with an average frequency λ_A . The nature of the track is indicated by a five-vector \vec{R} which contains the three position coordinates, the heading, and the velocity. We let the letter F designate friendly, H designate hostile, and N designate nonexistent tracks. The five-vector \vec{r} will designate the actual estimate of \vec{R} which is presented to the interception system. The error is then $\vec{\Delta e} = \vec{R} - \vec{r}$. In this manner, then, we speak of certain joint probabilities which express the reliability of the information presented to the interceptors:

$$P(\vec{R}, H | \vec{r}, H) \quad P(\vec{R}, F | \vec{r}, H) \quad P(\vec{R}, N | \vec{r}, H)$$

These are functions of the module decision processes discussed earlier, and the a priori probabilities of hostile, friendly and nonexistent tracks.

If the track is friendly or hostile, then four things can happen in the target-interceptor encounter. We let

DD	designate "both target and interceptor destroyed"
DS	designate "target only destroyed"
SD	designate "interceptor only destroyed"
SS	designate "neither destroyed."

If the target is nonexistent, we shall think of the encounter as "DS".

We will assume that friendly targets have the same defensive policies and capabilities as hostile targets. We also assume that, when the interceptor is not destroyed, there is a definite mission time $t(\vec{r})$, a fixed function of \vec{r} . When an interceptor is destroyed, we assume it is replaced thereafter in a time interval dt with probability ρdt . (This means that the probability it has been replaced t seconds after it should have returned is $1 - e^{-\rho t}$.)

We assume various probabilities for the results of interception. If the target is friendly or hostile, we have a set

$$P(DD|\vec{R}, \vec{r}) \quad P(DS|\vec{R}, \vec{r}) \quad P(SD|\vec{R}, \vec{r}) \quad P(SS|\vec{R}, \vec{r}).$$

Actually, the same question of whether or not the interceptor was destroyed can be answered by a probability distribution on the amount of time it takes to return, since the replacement of a destroyed interceptor can be regarded as the return of the interceptor. Thus the probability that an existent target is destroyed and the interceptor returns between t and $t + dt$ seconds after mission start, given \vec{R} and \vec{r} , is

$$P(\bar{D}, t|\vec{R}, \vec{r}) dt$$

and the probability that the target is not destroyed is

$$P(S, t|\vec{R}, \vec{r}) dt.$$

If the target is nonexistent, there is only

$$P(D, t|\vec{R}, \vec{r}) dt.$$

Since we are examining the interception node, we allow no possibility for an "attack" command being given when no interceptors are available. Also, because of the nature of the weapons in question, we will assume that a target which is not destroyed in the first encounter with an interceptor will not be destroyed at all.

There is a definite expected loss associated with the assignment of an interceptor to a particular target. If the target is friendly, then the loss is α times the probability of destroying that target. We associate a loss of 1 with a hostile target which is not destroyed either because interception failed or no attempt was made to intercept.

In particular, suppose that in a given time, N possible hostiles are at hand, n of which are attacked by the interceptors. The estimated coordinates of these targets at the time the attack commands were given are $\vec{r}_1, \vec{r}_2, \dots, \vec{r}_n$. In such a case, the estimated loss associated with the N targets is

$$\begin{aligned} & \sum_{i=n+1}^N \int_{\vec{R}} P(\vec{R}_i, H | \vec{r}_i, H) d\vec{R}_i + \alpha \sum_{i=1}^n \int_{\vec{R}} P(\vec{R}_i, F_i | \vec{r}_i, H) \int_0^{\infty} P(D, t | \vec{R}_i, \vec{r}_i) dt d\vec{R}_i \\ & + \sum_{i=1}^n \int_{\vec{R}} P(\vec{R}_i, H | \vec{r}_i, H) \int_0^{\infty} P(S, t | \vec{R}_i, \vec{r}_i) dt d\vec{R}_i. \end{aligned} \quad (13)$$

Of course, the analysis of the interception node must take into account the rule by which assignments of interceptors are made by modules M_5 and M_6 . Unfortunately, this rule will not be very simple, regardless of what assumptions are made, because certain types of tracks will have priority. Thus the problem is not that of simple servicing of equivalent situations. Certain types of tracks will not be attacked at all, for the probability of destroying them would be much less than the probability that a new track will appear while the interceptor is engaged.

We will define a function $A(\vec{r})$ which will fix the priorities on various tracks. When several tracks might be attacked, one would choose to attack the track for which $A(\vec{r})$ is maximum. The case $A(\vec{r}) = 0$ will indicate that the track is not to be attacked under any circumstances.

It should be emphasized that the priority rule, as a function on \vec{r} , which is the estimated coordinate vector at the time the assignment is made, appears to be the proper assignment criterion for the weapons. If track A has priority over track B, and B has priority over C, then it is clear that A has priority over C. While $A(\vec{r})$ is explicitly a function only of \vec{r} , it is implicitly a function of all governing statistical rules of the situation.

Another point bears emphasis. The coordinates \vec{r} and \vec{R} which have been referred to in this section designate the estimated and real coordinates of tracks at the time an assignment is to be made, i.e., when there are unassigned tracks present and an interceptor is available. It is understood, of course, that the actual target coordinates are continually changing.

It should also be remembered that the probability functions thus far discussed are insufficient to define $A(\vec{r})$. This is true because $A(\vec{r})$ must be influenced, in part, by the statistics with respect to changes in the number of tracks and changes in the coordinates of tracks.

To outline the additional information that is necessary to define $A(\vec{r})$, assume that the average rate of arrival of tracks to the weapons node from the identification node is λ_w , and that the probability of a new track occurring in time dt is $\lambda_w dt$. There is a probability distribution on \vec{r} , $q(\vec{r})$, which expresses the probabilities that the initial locations of the tracks are at \vec{r} . There must also be a rule to describe how \vec{r} changes for a particular track. This would be a Markov-type rule, $P[\vec{r}(t_2) | \vec{r}(t_1)]$.

Then, for a particular $A(\vec{r})$, and given the set $P(\vec{R}, H | \vec{r}, H)$; $P(\vec{R}, F | \vec{r}, H)$; $P(\vec{R}, N | \vec{r}, H)$ and the set $P(D, t | \vec{R}, \vec{r})$; $P(S, t | \vec{R}, \vec{r})$ it would then be possible, at least in theory, to calculate the following:

(Rate at which hostiles are not intercepted) + (Rate at which hostiles are intercepted but not destroyed) $\div \alpha$ X (Rate at which friendlies are intercepted and destroyed), which is the average rate of loss in the weapons assignment and interception nodes.

Of course, it is not possible to calculate these rates theoretically. The obvious machine-servicing type model is inapplicable here, even in an elaborated form, because of the probability law which governs the return of an interceptor for new assignment. The knowledge of the approximate time that interception will take, given \vec{r} and the assumption that the interceptor is not destroyed, is a vital consideration in the weapons assignment problem which should not be ignored by assuming some conventional servicing model in order to ease analysis.

To summarize this section, it is suggested that the proper rule for the weapons assignment module M_6 is a priority law, $A(\vec{r})$, which is applied to tracks identified as hostile whenever an interceptor is available. This priority law must be

established by simulation since it is a complex function of the statistics of the information presented to module M_w and the statistics of interception. The information presented to the weapons assignment node by the identification node is expressed by a set of probability densities:

$$P(\vec{R}, H | \vec{r}, H) \quad P(\vec{R}, F | \vec{r}, H) \quad P(\vec{R}, N | \vec{r}, H),$$

the average rate λ_w and the probability density $q(\vec{r})$. These express the frequency, initial location, and accuracies of the tracks identified as hostile which are presented to the weapons assignment node of operation. The rate of loss due to the weapons assignment and control functions of modules M_5 and M_6 is L_w , and is a function of these five entities.

2. Identification Node of Operation:

It has been shown how the quality of the information passing from the identification node to the weapons assignment and control modules actually determines an average rate of loss, L_w , associated with the interception process. This rate of loss is a function of the frequency, λ_w , with which the "hostile" notification is given, the probability distribution, $q(\vec{r})$, of the initial coordinates of the tracks identified as hostile, and the conditional probability distributions

$$P(\vec{R}, H | \vec{r}, H) \quad P(\vec{R}, F | \vec{r}, H) \quad P(\vec{R}, N | \vec{r}, H).$$

Since the operations of the identification node determine these factors, it is clear that the proper evaluation of the identification node must involve, at least in part, the loss rate L_w .

The other part of the loss rate pertinent to the identification node is L_I , which is the average rate at which "hostiles" are identified as "friendly" and allowed to pass as such. Thus the identification node seeks to minimize

$$L_I + L_w.$$

It is useful at this point to describe qualitatively how the three factors determining L_w influence L_w . The influence of λ_w is clear. The greater the frequency of the "hostile" signal to the weapons node, the greater L_w .

The probability distribution $q(\vec{r})$ determines, in essence, how much time is available for interception. Thus L_w will be less to the extent that $q(\vec{r})$ favors position coordinates which are near the outer boundaries of the defense sector.

The three conditional probability distributions $P(\vec{R}, H|\vec{r}, H)$; $P(\vec{R}, F|\vec{r}, H)$; and $P(\vec{R}, N|\vec{r}, H)$ express the accuracy of the information presented to the weapons node. To the extent that nonexistent targets are identified as hostile, L_w will increase due to wasted interception time. To the extent that friendly tracks are identified as hostiles, L_w will increase due to wasted interception time and destruction of friendly craft. Also, to the extent that \vec{r} differs from \vec{R} (this is beyond the control of the identification node), L_w will increase due to the lesser probability of successful interception.

To further define the hypotheses concerning the identification node, we assume that, when the identification procedure is started for a particular track, T_4 seconds are spent by module M_4 correlating that track with logged data, after which time a decision "hostile" is made if it is ever to be made for that track. This "constant time" constraint or goal (g_{T_4}) for module M_4 was discussed in the previous section.

It is necessary to distinguish two cases. These are the cases where the average frequency of arrival of tracks to the identification node, λ_I , is less than or equal to $1/T_4$, and the case where $\lambda_I > 1/T_4$.

In the case $\lambda_I \leq 1/T_4$, all tracks may be processed by the identification node on a "first come-first serve" basis, with only a priority rule to govern a choice among several tracks (we ignore the possibility that a track may not be processed at all due to high velocity; the priority rule would tend to favor such a track in any case). In such a situation, it is clear that L_I is accounted for only by the failure to identify hostiles as such upon examination. This can happen in two ways. First, the logged flight plan information itself may be incorrect, and may coincidentally correlate with a hostile track. Second, a hostile track might correlate more closely with correct logged information than the relevant friendly track. The former possibility is so improbable that it will be ignored; the latter is a real possibility, given intelligence on the part of the enemy.

Designate by \vec{r}_0 the estimated coordinates of a track when it is received by the identification node, and let the probability distribution of \vec{r}_0 be $Q(\vec{r}_0)$. Also

assume that there is a probability $\beta = \beta(\vec{r}_0)$ that a given track is hostile although it correlates with correct logged data. Then

$$L_I = (\lambda_I - \lambda_W) \int_{\vec{r}_0} \beta(\vec{r}_0) Q(\vec{r}_0) d\vec{r}_0 \quad (14)$$

is the rate of loss associated with identifying hostiles as friendly.

It has been remarked that the operations of the identification node influence the rate of loss L_W through three types of performance characteristics; $q(\vec{r})$, λ_W , and the set $P(\vec{R}, H | \vec{r}, H)$, $P(\vec{R}, F | \vec{r}, H)$, and $P(\vec{R}, N | \vec{r}, H)$. The latter two are determined by the nature of the information delivered by the surveillance node and the time T_4 spent in identification on a given track. Only the first characteristics, $q(\vec{r})$, which is the probability distribution of the tracks identified as hostile at the time this identification is made, is influenced by the priority rule in the identification node. The priority rule should be adjusted so that $q(\vec{r})$ is such that L_W is minimized. Expressed in simple, qualitative terms, this means that higher velocity tracks initiated at points from which attack is likely should receive priority so that sufficient time is allowed for interception.

Of course, it is not possible to develop the optimum priority rule by completely analytical means. It is a simple matter, however, to place a theoretical upper limit on $q(\vec{r})$ by assuming that the incidence of new tracks to the identification node is such that each track is processed immediately upon incidence. This situation is approached for $\lambda_I \ll i/T_4$, or for the condition $\lambda_I \ll n/T_4$, where n is the number of module M_4 identification channels. Let us define

$$P(H | \vec{r}_0)$$

as the probability that a track is identified as hostile, given its original location at \vec{r}_0 . Then

$$P(H, \vec{r}_0) = P(H | \vec{r}_0) Q(\vec{r}_0)$$

$$P(H) = \int_{\vec{r}_0} P(H | \vec{r}_0) Q(\vec{r}_0) d\vec{r}_0$$

So

$$P(\vec{r}_0 | II) = \frac{P(II, \vec{r}_0)}{P(II)} = \frac{P(H | \vec{r}_0) Q(\vec{r}_0)}{\int_{\vec{r}_0} P(H | \vec{r}_0) Q(\vec{r}_0) d\vec{r}_0} \quad (15)$$

The probability that a track identified as hostile originated in $d\vec{r}_0$ is

$$P(\vec{r}_0 | II) d\vec{r}_0.$$

We seek the probability that a track identified as hostile is in cell $d\vec{r}$ at the time of identification. Let us associate with each \vec{r} a vector $\Delta\vec{r}$ such that $\vec{r} - \Delta\vec{r}$ is the estimated coordinate vector for the track T_4 seconds before identification. Since \vec{r} contains velocity and bearing information, $\Delta\vec{r}$ is a deterministic function of \vec{r} . In this case,

$$q(\vec{r}) = P(\vec{r} - \Delta\vec{r} | II) = \frac{P(II | \vec{r} - \Delta\vec{r}) Q(\vec{r} - \Delta\vec{r})}{\int_{\vec{r}_0} P(H | \vec{r}_0) Q(\vec{r}_0) d\vec{r}_0} \quad (16)$$

We are also able to estimate λ_w under these conditions:

$$\lambda_w = \lambda_I P(II) = \lambda_I \int_{\vec{r}_0} P(H | \vec{r}_0) Q(\vec{r}_0) d\vec{r}_0 \quad (17)$$

The conditional probability $P(II | \vec{r}_0)$ is, we are reminded, the probability that a track which originated at \vec{r}_0 in the identification node is identified as hostile. This probability should be expressed in terms of more fundamental characteristics. Let $h(\vec{r}_0)$ be the probability that a track originating at \vec{r}_0 is hostile, $f(\vec{r}_0)$ be the probability that a track originating at \vec{r}_0 is friendly, and $n(\vec{r}_0)$ be the probability that a track originating at \vec{r}_0 is nonexistent. Obviously,

$$h(\vec{r}_0) + f(\vec{r}_0) + n(\vec{r}_0) = 1.$$

Let $p(\vec{r}_0)$ be the probability that a friendly target originating at \vec{r}_0 is identified as hostile. Then,

$$P(H|\vec{r}_0) = [1 - \beta(\vec{r}_0)] h(\vec{r}_0) + p(\vec{r}_0) f(\vec{r}_0) + n(\vec{r}_0) \quad (18)$$

where it is assumed that all nonexistent tracks are identified as hostile.

It is also possible to express the set $P(\vec{R}, H|\vec{r}, H)$; $P(\vec{R}, F|\vec{r}, H)$; and $P(\vec{R}, N|\vec{r}, H)$ in terms of more basic factors. We may assume that

$$\begin{aligned} P(\vec{R}, H|\vec{r}, H) &= P(\vec{R}|\vec{r}) P(H|\vec{r}, H) \\ P(\vec{R}, F|\vec{r}, H) &= P(\vec{R}|\vec{r}) P(F|\vec{r}, H) \\ P(\vec{R}, N|\vec{r}, H) &= P(\vec{R}|\vec{r}) P(N|\vec{r}, H). \end{aligned} \quad (19)$$

The probability that a track is nonexistent, given that it has been identified as a hostile at \vec{r} , can be taken to be $n'(\vec{r} - \Delta\vec{r})$. The probability that a track is friendly, given that it has been identified as a hostile at \vec{r} , will be designated $\pi(\vec{r} - \Delta\vec{r})$. Thus,

$$\begin{aligned} P(\vec{R}, H|\vec{r}, H) &= P(\vec{R}|\vec{r}) [1 - n'(\vec{r} - \Delta\vec{r}) - \pi(\vec{r} - \Delta\vec{r})] \\ P(\vec{R}, F|\vec{r}, H) &= P(\vec{R}|\vec{r}) \pi(\vec{r} - \Delta\vec{r}) \\ P(\vec{R}, N|\vec{r}, H) &= P(\vec{R}|\vec{r}) n'(\vec{r} - \Delta\vec{r}) \end{aligned} \quad (20)$$

$$\text{where, } \pi(\vec{r}) = \frac{p(\vec{r}) f(\vec{r})}{P(H|\vec{r})} \quad \text{and} \quad n'(\vec{r}) = \frac{n(\vec{r})}{P(H|\vec{r})} \quad (21)$$

We are now in a position to recapitulate the critical characteristics pertinent to the identification and weapons direction-interception nodes which establish the overall rate of loss of the system. This is under the assumption that the identification node has sufficient capacity to process each track immediately upon reception from the surveillance node. The critical characteristics are:

$Q(\vec{r}_0)$; the probability distribution of the track coordinates at the time they are initiated in the identification node.

$\beta(\vec{r}_0)$; the probability that a hostile track initiated at \vec{r}_0 is identified as friendly.

$h(\vec{r}_0)$; the probability that a track originating at \vec{r}_0 is hostile.

$h(\vec{r}_0)$; the probability that a friendly track initiated at \vec{r}_0 is identified as hostile.

$f(\vec{r}_0)$; the probability that a track originating at \vec{r}_0 is friendly.

$n(\vec{r}_0)$; the probability that a track at \vec{r}_0 is nonexistent.

$n(\vec{r}_0) = 1 - h(\vec{r}_0) - f(\vec{r}_0)$.

λ_I ; the rate of arrival of tracks to the identification system.

$P(\vec{R}|\vec{r})$; the conditional probability distribution of the true coordinates, given the observed coordinates. If we assume $P(\vec{r}) = P(\vec{R})$, then $P(\vec{R}|\vec{r}) = P(\vec{r}|\vec{R})$.

$P[\vec{r}(t_2)|\vec{r}(t_1)]$; a Markov-type rule to express the manner in which track coordinates change with time. For practical purposes, it may be permissible to express this in a deterministic manner, $\vec{r}(t_2)$ being the extrapolated value of $\vec{r}(t_1)$ with probability unity.

$P(D, t|\vec{R}, \vec{r})$; $P(S, t|\vec{R}, \vec{r})$; a set representing the results of interception.

Several of these entities will, in turn, be expressible in terms pertinent to the surveillance node. The factors $Q(\vec{r}_0)$, $n(\vec{r}_0)$, λ_I , and $P(\vec{R}|\vec{r})$, in particular, will be largely determined by the surveillance node, which will be examined in the next section.

3. Surveillance Node of Operation

It has been shown that losses may originate in the identification node due to failures to identify hostiles as such. Losses may also occur in the weapons assignment - interception node due to the failure to destroy hostile targets (either through failure to attempt interception or failure of interception) or to the successful destruction of friendly craft. The mathematical factors which determine the average rate of these losses have been outlined.

It is also possible for a loss to originate in the surveillance node when a hostile track is not designated as a track. Some mathematical preliminaries will be necessary before this loss can be defined mathematically.

The surveillance node will be assumed to consist of only the track initiator and track monitor modules M_1 and M_2 . The height finding module M_3 will be neglected since it performs a parallel function with M_2 . The function of M_1 was shown

to be that of determining from the radar returns which returns are tracks. Module M_1 was defined as determining the position coordinates of tracks passed to module M_2 .

The function of module M_2 was defined as making continuing estimates of the position, velocity, and direction coordinates of all tracks received from module M_1 .

It may be seen at this point that the operations of the surveillance node are of higher complexity than those of the identification node. There are two complicating features, 1) The surveillance node contains two distinct modules, M_1 and M_2 ; 2) While a definite processing time could be realistically assigned to the decision element of the track initiator module M_1 respecting whether or not a set of returns represents a track, it would not be in accord with the basic character of the operations of the track monitor module M_2 to assign such a fixed time.

It appears that the easiest way to treat the latter complication is to consider the track monitor module to be in turn divided into two sub-nodes. The first sub-node constructs the initial estimate of velocity and heading for a track received from the track initiator module; the second sub-node performs the continued monitoring of the tracks.

Let T_1 be the time required in the track initiator module, and let T_2 be the time required in the initial track monitor node. In each case, that is, we will assume that a fixed amount of time (fixed time goal) is spent on each operation. If we further assume that $T_1 \geq T_2$, then it is clear that, whatever the sequence of operations in the track initiator module, the optimum rule of operation for the initial track monitor node is "first come-first serve". This simplifying assumption will be made. This permits the virtual disregarding of the initial track monitor sub-node for purposes of modeling and analysis. We assign a time $T_s = T_1 = T_2$ necessary for the initiation and initial velocity and direction estimation of returns judged to represent a track, and a time T_1 necessary for the "no track" decision.

The sub-node which estimates velocity and bearing on a continuing basis is somewhat more elusive from the standpoint of optimum sequencing, but the associated problems yield when one considers the type of identification node assumed in the preceding section, i.e., one in which all tracks are processed immediately upon reception, and not reprocessed. In such a case, there is no point in continued monitoring of tracks not identified as hostile (since the loss will not be decreased by con-

tinued surveillance), so that the first rule for the sub-node which monitors on a continuing basis would be that only identified hostiles would be monitored.

It would appear at first thought that a priority rule for monitoring hostiles, similar to the weapons direction priority rule, is required for the optimum sequencing of the sub-node performing continual monitoring, but the situation can easily be more complex than this. This is because there are two basic classes of hostiles in general, those under attack and those not under attack. Depending upon the detailed nature of the weapons in question, it may be profitable to favor the monitoring of targets under attack, or it may be profitable to waive monitoring of targets upon which attack has commenced. The variables pertaining to a particular hostile track which must enter into the priority rule for this sub-node are (1) the coordinates \vec{r} extrapolated to present time, (2) the amount of time since the last velocity-direction estimate, (3) the question of whether or not it is under attack, and (4) the nature and coordinates of the weapon making the attack.

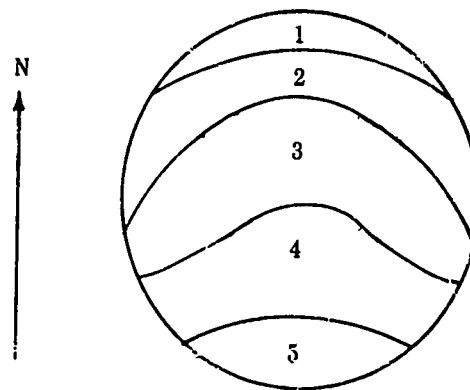
Because of the complexity of this sub-node, and because this sub-node is not part of the essential direct line of surveillance-identification-weapons direction, but only serves to refine old information, it will be disregarded in this mathematical formulation. It should be suggested, parenthetically, that any modifications of the present SAGE system in the direction of completely automatic operation should include automating at least the track monitor function.

This discussion shows that, under the conditions and assumptions stated, the surveillance node may be thought of as a single node, in which T_1 seconds are required for the "no track" or "track" decision. The "track" decision also yields the position coordinates immediately. The initial estimate of velocity and bearing is, strictly speaking, made in a separate sub-node, but if the number of track monitor channels is equal to or greater than the number of track initiator channels, and if the time required for initial monitoring $T_2 \leq T_1$, then the initial track monitor sub-node contributes only a time delay to the passage of a track from the track initiator module M_1 to the identification module M_4 .

With respect to the sequencing problem in the track initiator module M_1 , it could be argued that there is no reason for not specifying a "first come-first serve" rule for sets of returns, for the reason that nothing is supposedly known about such sets (except existence) at this point. Actually, this is not true, for the present physical mechanization makes two space coordinates at least approximately known

at the outset of processing, so there is a basis upon which to construct a priority rule.

The priority rule must have, in fact, a particularly close relationship to the specific physical mechanization in this case, i.e., it must be cognizant of the present situation in which a human observes the situation display (Figure 11) and is necessarily made aware of the approximate values of two position coordinates in the process of selecting a region of returns to process. This implies that the priority rule should be based on the location of returns in discrete "cells" on the situation display, as depicted below.



The screen is assumed to depict the sky in an area in which attacks are likely to originate from the north in order to destroy the city in the south. In such a case, a return in area 1 would have priority over a return in area 2, and so forth.

It is not being asserted that the display division must necessarily be as shown here; the screen divisions must ultimately be based upon the average rate of loss. Only the general form of the appropriate priority rule is being suggested.

It should be remembered that the divisions of the display, just as the weapons assignment priority rule, are functions of the particular statistics of aircraft and system operations assumed at the time. Therefore, it is suggested that the lines defining the display sectors be contained on masks which are replaceable as statistics change.

With these extensive preliminary remarks in mind, the basic mathematical characteristics of the surveillance node can now be defined.

Assume that

λ_H = rate of arrival of signals representing hostile tracks to the initiator module M_1 .

λ_F = rate of arrival of signals representing friendly tracks to the track initiator module M_1 .

λ_N = rate of arrival of signals representing noise to the track initiator module M_1 .

Assume also that the probability distribution of hostile signals when they commence in track initiation is $H(\vec{R})$, that the probability distribution of friendly signals when they originate in track initiations is $F(\vec{R})$, and that the probability distribution of returns representing noise, when they originate in track initiation, is $N(\vec{R})$.

Assume that $\theta = \epsilon_{11}$ is the probability that a track is identified as noise, and that $\phi = \epsilon_{10}$ is the probability that noise is identified as a track. This was discussed in an earlier section. Then the rate of loss originating in the surveillance node is

$$\theta \lambda_H$$

and the rate at which the identification node receives the "track" notification is

$$\lambda_I = (1 - \theta) (\lambda_F + \lambda_H) + \phi \lambda_N. \quad (22)$$

One is also able to calculate $Q(\vec{r}_0)$, the probability distribution of the track coordinates at the time they are initiated in the identification node. We associate a $\Delta \vec{R}$ with \vec{R} , such that \vec{R} is the value of $\vec{R} - \Delta \vec{R}$ extrapolated T_s seconds in the future. Then,

$$Q(\vec{R}) = \frac{(1 - \theta) \lambda_H}{\lambda_I} H(\vec{R} - \Delta \vec{R}) + \frac{(1 - \theta) \lambda_F}{\lambda_I} F(\vec{R} - \Delta \vec{R}) + \frac{\phi \lambda_N}{\lambda_I} N(\vec{R} - \Delta \vec{R}) \quad (23)$$

where again it has been assumed that the track initiator module processes each set of returns immediately upon reception, and that $Q(\vec{R})$ is a close approximation to $Q(\vec{r}_0)$.

It is also possible to express $h(\vec{R})$, $f(\vec{R})$, and $n(\vec{R})$. If a track originates at \vec{R} in track initiation and is designated as a track, then the probability that it is hostile is

$$\frac{(1 - \theta) H(\vec{R})}{(1 - \theta) H(\vec{R}) \frac{\lambda_H}{\lambda_H + \lambda_F + \lambda_N} + (1 - \theta) F(\vec{R}) \frac{\lambda_F}{\lambda_H + \lambda_F + \lambda_N} + N(\vec{R}) \phi \frac{\lambda_N}{\lambda_H + \lambda_F + \lambda_N}} \times$$

$$\frac{\lambda_H}{\lambda_H + \lambda_F + \lambda_N} = \frac{\lambda_H (1 - \theta) H(\vec{R})}{(1 - \theta) [\lambda_H H(\vec{R}) + \lambda_F F(\vec{R})] + \phi \lambda_N N(\vec{R})} \quad (24)$$

and the probability it is friendly is

$$\frac{\lambda_F (1 - \theta) F(\vec{R})}{(1 - \theta) [\lambda_H H(\vec{R}) + \lambda_F F(\vec{R})] + \phi \lambda_N N(\vec{R})}$$

and the probability it is noise is

$$\frac{\phi \lambda_N N(\vec{R})}{(1 - \theta) [\lambda_H H(\vec{R}) + \lambda_F F(\vec{R})] + \phi \lambda_N N(\vec{R})}$$

Thus, one can say that

$$h(\vec{R}) = \frac{(1 - \theta) \lambda_H H(\vec{R} - \Delta\vec{R})}{(1 - \theta) [\lambda_H H(\vec{R} - \Delta\vec{R}) + \lambda_F F(\vec{R} - \Delta\vec{R})] + \phi \lambda_N N(\vec{R} - \Delta\vec{R})} \quad (25)$$

$$f(\vec{R}) = \frac{(1 - \theta) \lambda_F F(\vec{R} - \Delta\vec{R})}{(1 - \theta) [\lambda_H H(\vec{R} - \Delta\vec{R}) + \lambda_F F(\vec{R} - \Delta\vec{R})] + \phi \lambda_N N(\vec{R} - \Delta\vec{R})} \quad (26)$$

$$\vec{n}(\vec{R}) = \frac{\lambda_N \phi_N(\vec{R} - \vec{\Delta R})}{(1 - \theta) \left[\lambda_H H(\vec{R} - \vec{\Delta R}) + \lambda_F F(\vec{R} - \vec{\Delta R}) \right] \phi \lambda_N N(\vec{R} - \vec{\Delta R})} \quad (27)$$

As suggested in the preceding section, the conditional probability density $P(\vec{R}|\vec{r}) = P(\vec{r}|\vec{R})$ if one assumes $P(\vec{r}) = P(\vec{R})$. In this case, $P(\vec{R}|\vec{r})$ is a function only of the quality of radar information and the operations of the surveillance node. One should also note that the condition for sufficiency of a single surveillance channel, under the assumption $T_1 \geq T_2$, is

$$\lambda_H + \lambda_F + \lambda_N \leq \frac{1}{T_1} \quad (28)$$

4. Summary and Conclusions

The basic mathematical characteristics of the SAGE system are best summarized with reference to Figure 19, Figure 20, and Figure 21. Figure 19 shows the nature of the input information to the surveillance node, the factors determined by the operations of this node, the loss associated with the node, and the nature of the output information from this node. By the "nature" of the input and output characteristics is meant, of course, the probabilistic laws governing this information. The actual meaning of the information is clear from consideration of the node; the output of the surveillance node is "track" and " \vec{r} ". The factors appearing in Figure 19 are summarized as follows:

- λ_H ; the average frequency of arrival to the surveillance area of hostile tracks.
- λ_F ; the average frequency of arrival to the surveillance area of friendly tracks.
- λ_N ; the average frequency of arrival to the surveillance node of sets of returns representing noise.
- $H(\vec{R})$; the probability density function of the coordinate of hostile tracks as they initiate in the surveillance area.
- $F(\vec{R})$; The probability density function of the coordinates of friendly tracks as they initiate in the surveillance area.

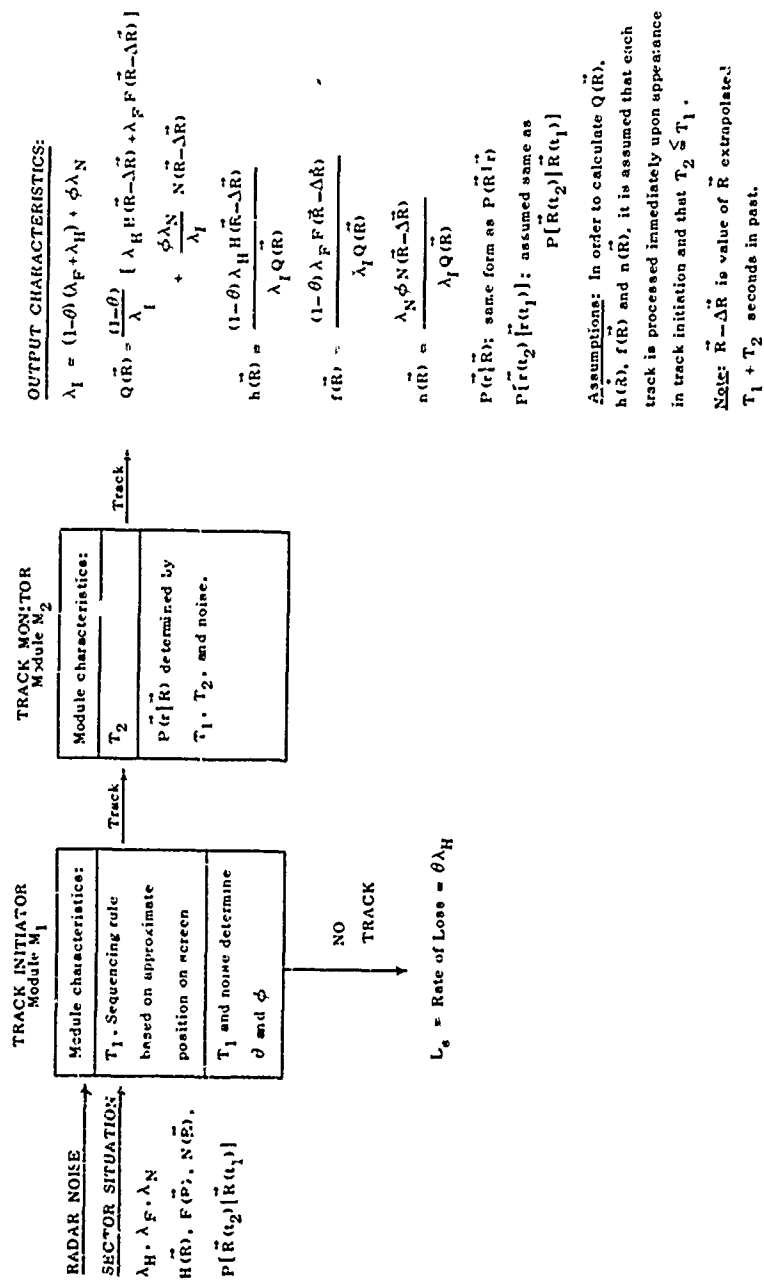


Figure 19. Surveillance Node

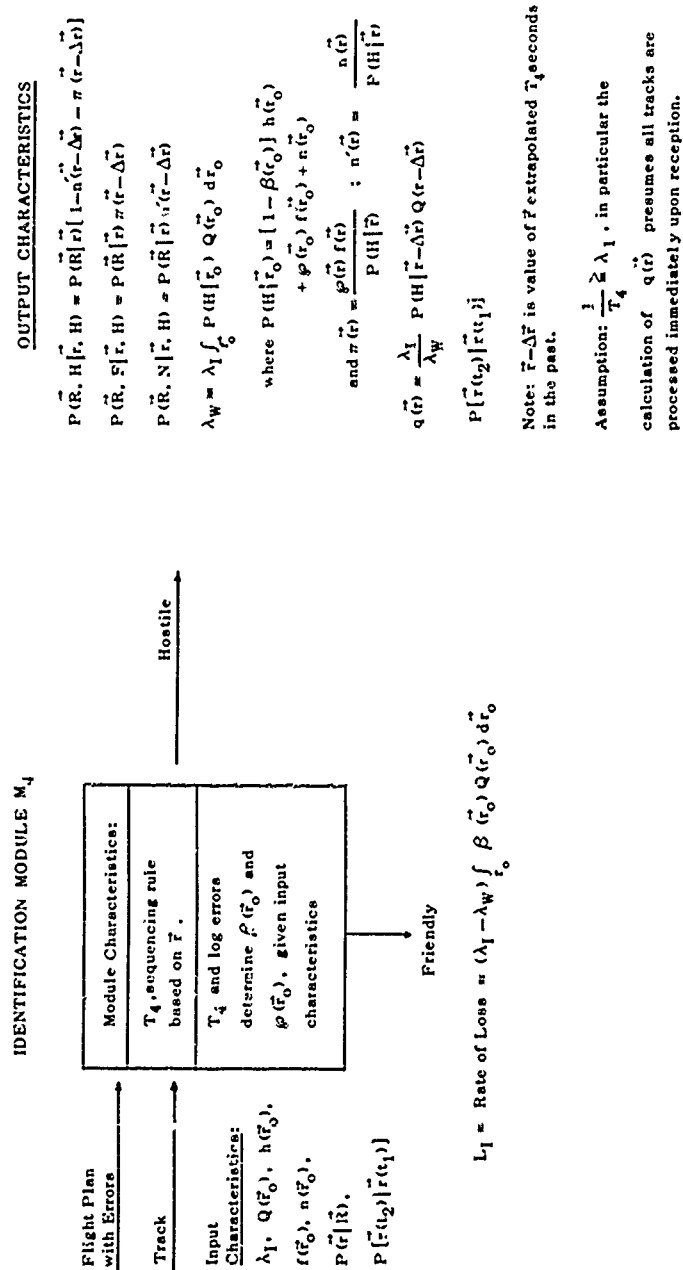
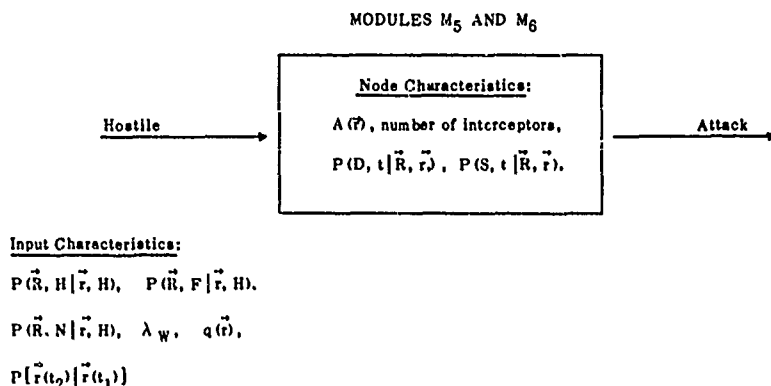


Figure 20. Identification Node



Rate of Loss = L_w = Rate at which hostiles are not intercepted + Rate at which hostiles are intercepted but not destroyed + $\alpha \times$ (Rate at which friendlies are intercepted and destroyed).

Figure 21. Weapons Direction-Interception Node

- $N(\vec{R})$; the probability density function of the coordinates of noise returns as they initiate in the surveillance node.
- $P[\vec{R}(t_2) | \vec{R}(t_1)] d\vec{R}(t_2)$; the probability that a track is in cell $d\vec{R}(t_2)$ in coordinate space at time t_2 , given its location at $\vec{R}(t_1)$ in coordinate space at time t_1 .
- T_1 ; the time spent on each track by track initiator module M_1 .
- θ ; the probability that a track is identified as noise.
- ϕ ; the probability that noise is identified as a track.
- L_s ; the rate of loss in the surveillance node.
- T_2 ; the time spent on each track in the initial track monitor module sub-node.
- $P(\vec{r} | \vec{R}) d\vec{r}$; the probability that the coordinate estimate for a track is in $d\vec{r}$ in coordinate space, given true coordinates \vec{R} .
- λ_I ; the rate at which the "track" notification is given to the identification node.
- $Q(\vec{R})$; the probability density function of the coordinates of tracks as they commence in the identification node.

- $\vec{h}(\vec{R})$; the a priori probability that a track is hostile, given its initiation in the identification node at \vec{R} .
- $\vec{f}(\vec{R})$; the a priori probability that a track is friendly, given its initiation in the identification node at \vec{R} .
- $\vec{n}(\vec{R})$; the a priori probability that a "track" initiated at \vec{R} in the identification node is actually noise.

Given the probability characteristics of the input information, it will be noted that only three factors, θ , ϕ , and $P(\vec{r}|\vec{R})$ depend on the human (and hence not analytical factors of the surveillance node).

The assumption that each set of returns is processed immediately upon initiation is made only to formulate the probability functions $Q(\vec{R})$, $\vec{h}(\vec{R})$, $\vec{f}(\vec{R})$, and $\vec{n}(\vec{R})$. If $1/T_1 > \lambda_H + \lambda_F + \lambda_N$, and $T_1 > T_2$, the result is a good approximation in any case.

Identification Node.

The characteristics of the identification node are summarized in Figure 20. The variables are:

- T_4 ; the amount of time spent processing each track.
- $\beta(\vec{r}_0)$; the probability that a hostile track initiated at \vec{r}_0 is identified as friendly.
- $\rho(\vec{r}_0)$; the probability that a friendly track initiated at \vec{r}_0 is identified as hostile.
- L_I ; rate of loss associated with identification node.
- λ_w ; rate of "hostile" notification to weapons direction.
- $P(\vec{R}, H | \vec{r}, H) d\vec{R}$; the probability that a track identified as a hostile at \vec{r} is a hostile in cell $d\vec{R}$ in coordinate space.
- $P(\vec{R}, F | \vec{r}, H) d\vec{R}$; the probability that a track identified as a hostile at \vec{r} is a friendly in cell $d\vec{R}$ in coordinate space.
- $P(\vec{R}, N | \vec{r}, H) d\vec{R}$; the probability that a track identified as a hostile at \vec{r} is noise in cell $d\vec{R}$ in coordinate space.

$\vec{q}(\vec{r})$; the probability density of the estimated coordinates of tracks identified as hostile at the time of identification (initiation in weapons direction).

Again, the assumption that all tracks are processed in the identification node immediately upon reception has been made in order to estimate the probabilistic character of the output information. This is a good approximation if $\lambda_1 < 1/T_4$.

Weapon Assignment Node.

Figure 21 summarizes the characteristics of the weapons direction-interception node. The basic characteristics are:

- $A(\vec{r})$; the priority rule on the coordinates. If an interceptor is available, the weapons node commands an attack on the track for which $A(\vec{r})$ is maximum, except that $A(\vec{r}) = 0$ indicates no attack.
- $P(D, t | \vec{R}, \vec{r}) dt$; the probability that a track with estimated coordinates \vec{r} and real coordinates \vec{R} at commencement of attack is destroyed and the interceptor returned in time interval $t, t + dt$.
- $P(S, t | \vec{R}, \vec{r}) dt$; the probability that a track with estimated coordinates \vec{r} and real coordinates \vec{R} at commencement of attack is not destroyed and the interceptor is returned in time interval $t, t + dt$.
- L_w ; rate of loss associated with the weapons direction-interception node. The overall rate of loss for the system, of course, is $L_s + L_i + L_w$.

This summary shows that extensive mathematical sensitivity analysis is possible in the SAGE system. In the surveillance and identification nodes it is possible to write mathematical expressions for the controlling characteristics of the output information in terms of the assumed input characteristics and a small number of factors which depend, at least partially, on the human operators (and therefore must be determined by simulation). The critical assumption which permits such extensive analysis is that each track is processed immediately upon incidence in each of the two nodes. That is, a strong time constraint exists. In the case where the capacity of each of these nodes is greater than the average rate of arrival of tasks, this assumption permits close theoretical approximations to the important probabilistic characteristics of the outputs. It follows, then, that to the extent the surveillance

and identification node capacities are sufficient for the average rates of arrival of tasks, the priority rule for these nodes has only second order importance, for it influences no important quantity or factor strongly.

The situation presented by the weapons direction-interception node is quite different. First, no important factor can be determined analytically. Second, the node characteristics are somewhat more complex (even assuming a given number of one type of interceptor of the variety that is expected to return after an attack). Third, the priority rule, $A(\vec{r})$ is a first order factor in this node.

The priority rule in the weapons direction-interception node is, as it is in other nodes, a deterministic function on the information about a particular track which has been gathered at that point. In a situation in which one of several tracks must be chosen for attack, one chooses that for which the priority function on the coordinates is maximum. This is not a sequencing rule in the normal sense; the sequencing is a result of application of the rule, not the rule itself.

All that is necessary to justify the propriety of such a priority rule is to agree that, for given statistics concerning tracks identified as hostile, one would attack a in a situation involving one interceptor and identified hostiles a, b, and c if one would attack a in a situation a, b and c in a situation b, c.

It must be remembered that all priority rules in SAGE, particularly $A(r)$ are determined for a given set of system and disturbance statistics, although they are explicitly functions only of information pertaining to particular tracks. Changes in statistics imply changes in the priority rule.

It is believed that any future mathematical sensitivity analysis of SAGE involving the pertinent figures of merit should emphasize the complex servicing problem presented by the weapons direction-interception node.

B. MODULE PROCESSING RATE SENSITIVITY MODELS.

The previous section developed the sensitivity analysis of the system for time constraints for each of the modules. The loss functions were developed for the decision errors. Since the processing rates of the modules were assumed to be high with respect to the rate at which tracks entered the nodes of operation, loss functions due to track delays were assumed small and treated in the weapons node of operation. The loss due to the number (L) of tracks waiting to enter the system was assumed to be small due to the high input processing rate, $1/T_1$, by module M_1 .

$$\lambda = \lambda_H + \lambda_F + \lambda_N \leq \frac{1}{T_1} \quad (29)$$

Decision errors in high noise conditions can be reduced by allowing more processing time (minimum error goal) by the modules. However, when the input rate λ is high, delays now result and blocking and cross-coupling effects become significant. It was stated earlier that the system criterion is affected by such delays. Therefore, loss functions must be developed for the decision delays in addition to decision error loss functions. These delays must be cognizant of the blocking effects of the modules.

The establishment of delay loss functions is a difficult problem because now loss and priority functions must be developed for different types of tracks. However, it was seen from Figure 18 that the primary delay and blocking effects can occur at the input modules M_1 and M_2 . This is because of the monitoring function that module M_2 performs for the rest of the system modules M_4 , M_5 and M_6 . Therefore, any delay effects of these modules indirectly affects module M_2 's operation. In addition, radar noise conditions affect the operations of modules M_1 and M_2 which further influences the delays of tracks entering the system. Therefore, a measure of the input delays might be obtained by the number of tracks (L) waiting to be processed.

The purpose of this section will be to model module M_1 's operation when processing incoming tracks and relate its processing rate to the number of tracks waiting, L . In addition, the blocking effects between modules M_1 and M_2 are modeled and the combined processing rate of these modules is also related to L .

Due to the varying radar noise conditions on each track as it enters the system and the probabilistic nature of track arrivals, any modeling of the processing rates of modules M_1 and M_2 must be performed in probabilistic terms.

1. Model for Track Initiator Module M_1 .

The rate at which tracks can be processed by module M_1 is dependent upon

- (1) Average detection rate (μ_1) of decision element
- (2) Average processing rate (μ_c) of computer element
- (3) Average decision rate (μ_2) of decision element.

Figure 12 showed a diagram of these rates which are necessary for transforming the information state of a track from a "possible" state H_B to a "velocity" state H_A by module M_1 .

The expression for the expected number of tracks, L , waiting to be processed by module M_1 is developed as a function of the input rate λ and μ_1 , μ_c , and μ_2 . The condition between these rates, which assures that the expected number of tracks L will not diverge, is also developed.

The reader will note that, while constant service times were assigned to the various elements in the preceding section, these times are described in the probabilistic terms here. The real situation, of course, is best described in probabilistic terms much more complex than those assumed here. The constant service time approximation is felt to be quite close, and has the merit of permitting facile computation of module input - output error relationships. The statistical formulation presented here shows, in principle, how one may approach analysis of the real case which, however, would require computer solutions because of the time varying characters of the μ factors.

The approach is to consider module M_1 as capable of performing both the position and velocity-direction transformations on a track. The logic of separating these two transformations among the two separate modules M_1 and M_2 is then discussed.

The model development will consider module M_1 in one of a number of possible states (n, i) . These states will be defined as the number n of tracks waiting to be detected and/or processed by the decision element of module M_1 . The module itself will be defined as being in one of three possible states, designated as a , b , or c . These states will be discussed in the following paragraphs.

By knowing the probability of transition between states, a set of linear, constant coefficient, first order differential equations can be developed relating the state probabilities to the average track arrival rate λ and the average processing rates of the decision and computer elements (μ_1, μ_c, μ_2) .

a. Poisson Track Input Distribution

It is assumed that the probability P_n of n tracks arriving in an interval of time Δt follows a Poisson distribution; that is,

$$P_n(\Delta t) = \frac{(\lambda \Delta t)^n}{n!} e^{-\lambda \Delta t} \quad (30)$$

where

n - Number of tracks which arrive in an interval Δt , and

λ = Average arrival rate of tracks per unit time.

Then, as Δt approaches dt ,

$$P_n(dt) \approx \frac{(\lambda dt)^n}{n!} \quad (31)$$

There is a rational basis for using the Poisson process to describe the inputs to the modules. In evaluating the processing rate requirements of the input module, we can be quite arbitrary with what input distribution we choose. In the case of the other modules, the Poisson process may be looked upon as a limit of disintegration of scheduled processes. As each track is processed by a module as it flows through the system, the processing times will vary because of the many probabilistic factors. This tends to randomize the inputs to the following modules. This is an intrinsic characteristic of the Poisson Law.

Studies were made of the NORAD air defense system to determine the times when unknown tracks were reported and when the report was terminated.⁽¹⁹⁾ Estimates were made for 96 instances in every day at regular 15-minute intervals. Average values for two-hour periods were computed by combining the 15-minute interval estimates. The peaks for unknown tracks coincided closely with peaks in air traffic. There was also a definite cycle over the days of the week. The number of unknowns on the plotting board at any moment could be satisfactorily represented by a Poisson distribution.

b. Decision Element Assumptions

The following are the assumed functions of the decision element of module M_1 :

1. Detection function - Scanning the display and detection of new possible tracks.

2. Order function - Ordering the possible detected tracks in an optimum sequence for further processing. (Such optimum sequences were discussed in the previous section and will also be discussed in the following section.)
3. Request function - Any actions which are required to request or set up the computer element for an information request.
4. Decision function - Reviewing the completed information request from the computer element via the display element and making two decisions. The first decision is a track or no track decision. The second decision transforms the information state of a track into its position, velocity, and direction coordinates.

The sequence of functions of the decision element are then:

Detect - Order - Request - Decision

c. Module States

The possible states of the module can now be defined as a, b, or c.

(1) State (n, a)

The decision element is performing either the detection or ordering functions, and n tracks are waiting to be detected and/or processed.

The decision element will be assumed to scan the display to detect new input stimuli which are possible tracks. These possible tracks may have arrived during the time interval when the decision element was processing another track and thus was unable to perform the detection function. The detection process involves observing the radar returns in a given region of the display. The radar return history information is observed for a period of time before the position of a possible track can be determined (Figure 14).

Once a number of possible tracks have been detected, the decision element may spend a time interval resolving which track to further process. This is the ordering function.

Due to different noise conditions, some tracks are assumed to take longer to detect in this state than others. Some tracks may take no detection time if they enter under noise-free conditions.

It is assumed that the time the decision element spends in this state is random and follows an exponential service time density function, namely,

$$s_1(t) = \mu_1 e^{-\mu_1 t} \quad (32)$$

where

$s_1(t)dt$ = probability that the decision element will perform the above functions and request the computer to process a track at time t , and

$\mu_1 dt$ = probability that only one request will be made. Here, μ_1 is the average rate of requests per unit of time.

(2) State (n, b)

The decision element will have performed the request function and the computer element will be processing the information request on the track, and n tracks are waiting to be detected and/or processed.

The request function requires the rough estimation of the velocity vector (magnitude and direction) of the possible track. This estimate is performed by the decision element (man) and set up in the computer element as the initial condition for the following computations. It is assumed that this request function was completed in state (n, a).

The computer element in state (n, b) performs the following computations on the information request:

1. Radar coordinate conversion
2. Correlation of radar data with track
3. Processing of correlated data to reject clutter and detect accelerations of track.
4. Use of position information to predict and smooth track velocity and position.

After the decision element has requested the computer to process the above tracking problem, a delay may result. This delay is a function of the computer program sequence logic, the priority of the request and how many higher priority requests exist from other decision elements (men) in the system. Each module in the system will have a different priority index depending upon the priority of its function with regard to the system criterion.

It is assumed that the time delay in this state is random and follows a uniform time density function. It is also assumed that the delay in this computer state is small with respect to the delays in states a and c.

It should be noted that the computer delay effects on each module's operation would be included here in the modeling of the central computer operation when these delays are significant.

(3) State (n, c)

The decision element will be reviewing the information request from the computer element and making one of the two decisions involved in the decision function, and n tracks are waiting to be detected and/or processed. This represents the decision state of the module. In this state it is assumed that the computer has finished the problem or information request. For modules M_1 and M_2 , this information request is the calculation of velocity and direction of the possible track.

In this state the decision element must make two decisions. The first is the decision whether or not the stimuli, on which it requested computation, is a track. If the magnitude of the velocity vector proceeds to zero on the display, the decision element makes the decision "no track." If the velocity information does not go to zero then the decision is "track."

The second decision determines the position, velocity, and direction information state of the track. The decision element will be assumed to make this final information state decision when the accuracy is sufficient for identification of the track by the identification module M_4 . The decision element may spend a period of time monitoring the track through regions of noise until sufficient accuracy is achieved. This was discussed under the functions of module M_2 in a previous section.

Once a level of accuracy is achieved, the decision element will make the final decision as to the information state of the track.

The delay in this state is assumed to be a function of the noise conditions (for a fixed level of accuracy) and follow an exponential service time density function:

$$s_2(t) = \mu_2 e^{-\mu_2 t} \quad (33)$$

where

$s_2(t) dt$ = probability that the decision element will make a decision on the information state of the track at time t , and

$\mu_2 dt$ = probability that only one decision will occur. Here, μ_2 is the average rate of decisions per unit time.

d. Processing of Next Track

At the end of state (n, c) the decision element of module M_1 sends the identification module M_4 the position, velocity, and direction coordinates of the track. (The height coordinate is obtained by the height module M_3 .) After this notification the decision element of module M_1 returns to state (n, a) to start scanning for new tracks which may have arrived during states (n, b) and (n, c). The cycle then repeats itself. Each transition out of state (n, a) reduces the number L of tracks waiting by one.

e. Mathematical Model

With the preceding state definitions, and the assumption of exponential service and arrival times, the Markov process can be utilized as a mathematical representation of module M_1 . The Markov process describes the discrete states of the module continuously with respect to time.

In general, transition probabilities can be functions of the time interval since the last change of state. That is, the transition probability may be dependent upon the time since the last track input to module M_1 . Only when the arrival and service distributions are exponentially distributed can this time dependence be eliminated.

The following definitions are made:

$T_{i,n}(dt)$ = the transition probability that the module will change from state (i) to the nth state during the interval of time dt, $i \neq n$

$T_{n,n}(dt)$ = the transition probability that the module will remain in the nth state during the interval of time dt

$P_n(t)$ = the probability that the module is in state n at time t, where n is the number of tracks waiting to be processed.

The probability of the module being in state n at time $t + dt$ is equal to the sum of the transition probabilities to the nth state plus the probability of remaining in the nth state. All these transition probabilities are weighted by the probability of being in the source state. This can be written in terms of the following balance equation:

$$P_n(t + dt) = T_{n-1,n}(dt) P_{n-1}(t) + T_{n,n}(dt) P_n(t) + T_{n+1,n}(dt) P_{n+1}(t) \quad (34)$$

where

$P(n, a) = P_{n,a}(t)$ = the probability that the module is in state a at time t and has n tracks waiting

$P(n, b) = P_{n,b}(t)$ = the probability that the module is in state b and has n tracks waiting

$P(n, c) = P_{n,c}(t)$ = the probability that the module is in state c and has n tracks waiting

$\lambda dt = T_{n-1,n}(dt)$ = the probability of one track arriving

$\mu dt = T_{n+1,n}(dt)$ = the probability of one transition out of state n + 1

$T_{n,n}(dt) = (1 - \lambda dt)(1 - \mu dt)$ = the probability that the module will not change its present state

λ = the average arrival rate of tracks and noise stimuli

μ_1, μ_c, μ_2 = the average service rates of the decision and computer elements, where it is assumed that $1/\mu_c = 0$. That is, the time delay in state (n, b) is zero.

The balance equation of the module in Figure 12 can now be written as:

$$\frac{d}{dt} P(n, a) = \lambda P(n-1, a) - (\lambda + \mu_1) P(n, a) + \mu_2 P(n, c)$$

$$\frac{d}{dt} P(n, c) = \lambda P(n-1, c) - (\lambda + \mu_2) P(n, c) + \mu_1 P(n+1, a). \quad (35)$$

These equations of detailed balance are stated in terms of the module's state probabilities. The time dependence indicates that under nonsteady state conditions, the state of the module is a function of time. Figure 22 shows a Markov state representation of the module's states.

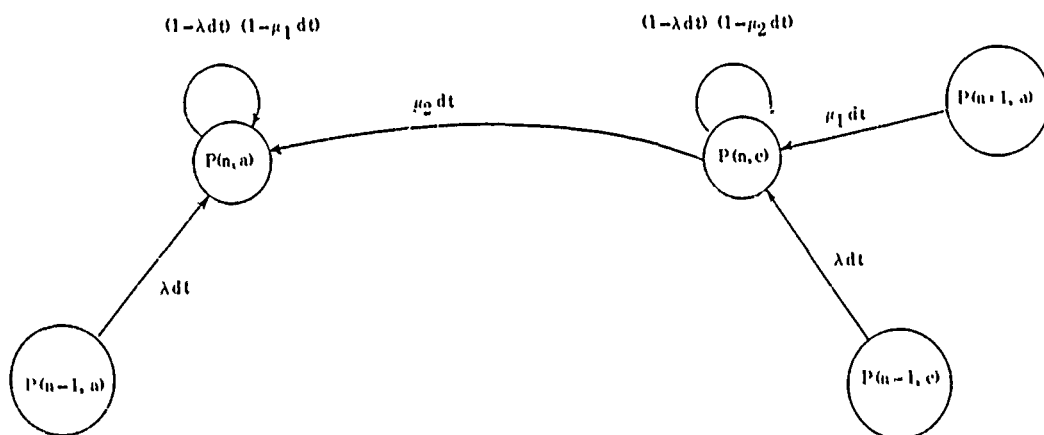


Figure 22. Markov State Diagram of Module M_1

In writing the balance equations, we are interested in two characteristics of the module. First, the number of tracks L which can be expected to be waiting to be processed under steady state conditions is desired. This gives a measure of the input processing rate capability of the module. This number cannot be large or many tracks will eventually leave the system and never be processed.

The second characteristic is the conditions on the arrival and processing rates (λ, μ_1, μ_2) which assure that the above expected number of tracks (L) will not diverge but reach a steady state. This gives a measure of the system stability.

Equation 35 under steady conditions becomes:

For $n \geq 1$,

$$\begin{aligned}\lambda P(n-1, a) - (\lambda + \mu_1) P(n, a) + \mu_2 P(n, c) &= 0 \\ \lambda P(n-1, c) - (\lambda + \mu_2) P(n, c) + \mu_1 P(n+1, a) &= 0.\end{aligned}\quad (36)$$

For $n = 0$, the initial conditions for these equations become

$$\begin{aligned}- (\lambda + \mu_1) P(0, a) + \mu_2 P(0, c) &= 0 \\ - (\lambda + \mu_2) P(0, c) + \mu_1 P(1, a) &= 0.\end{aligned}\quad (37)$$

Since we are interested only in the buildup of the number of tracks, let $P_i(n)$ stand for any of the two steady state probabilities $P(n, a)$ and $P(n, c)$.

The determinantal equation for equation 37 becomes:

$$\begin{vmatrix} 1 - (1 + \frac{1}{\rho_1}) E & \frac{1}{\rho_2} E \\ \frac{1}{\rho_1} E^2 & 1 - (1 + \frac{1}{\rho_2}) E \end{vmatrix} P_i(n) = 0 \quad (38)$$

where, E = the raising operator, and

$$\rho_i = \lambda / \mu_i = \text{utilization ratios.} \quad (39)$$

The roots of the above determinantal equation become:

$$r_i = 1, r_2, r_3 = \frac{(\rho_1 \rho_2 + \rho_1 + \rho_2) \pm \sqrt{(\rho_1 \rho_2 + \rho_1 + \rho_2)^2 - 4 \rho_1 \rho_2}}{2} \quad (40)$$

According to standard techniques in the theory of difference equations, the solutions for $P_i(n)$ are given by linear combinations of the n th powers of the roots of the determinantal equation; that is,

$$P_i(n) = \sum_j^3 c_{ij} r_j^n \quad i = 1, 2. \quad (41)$$

The mean number of tracks (L) which can be expected to be waiting in queue to be detected and/or processed by module M_1 becomes:

$$L = \sum_{n=0}^{\infty} (n+1) P_i(n). \quad (42)$$

The condition that assures that the expected number of tracks will not diverge is for each of the roots to be less than or equal to one ($r_j \leq 1$). It can be shown from equation 40 that this condition is satisfied when the following relations exist between λ , μ_1 , and μ_2 :

$$\rho_1 + \rho_2 = 1 \quad \text{where, } \rho_i = \lambda/\mu_i. \quad (43)$$

The above condition for convergence assures that the decision rates of the decision element (μ_1 , μ_2) are sufficient to prevent the number of tracks in queue from growing infinitely long (in infinite time).

The specification of the limit for the mean number L of tracks gives the processing rate requirements of module M_1 . This limit is a complex function of track types, priority functions, and delay loss functions as stated earlier.

When radar noise conditions increase and input rates increase, the requirement of another module M_2 to perform the monitoring function of state (n , c) becomes apparent. However, now blocking effects can occur between the modules. The development of the input processing requirements must be cognizant of such blocking effects.

2. Blocking Model of Modules M_1 and M_2 :

In the last section it was assumed that module M_1 performed the information state transformation of a track in state (n , c). It is apparent that if this transformation process was given to another module to perform, module M_1 could be free to perform only the detection function in state (n , a).

The addition of the track monitor module M_2 between modules M_1 and M_4 performs this function. However, another problem now arises which demonstrates how blocking effects may exist between modules in the system and also reduce the processing rate capability of the system.

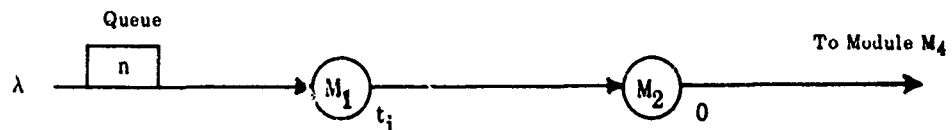
The blocking effect is caused by radar noise. It prevents module M_1 from returning to the detection state (n, a) until module M_2 has completed processing of its track.

a. Mathematical Model

The blocking effect between modules M_1 and M_2 can be modeled in a similar fashion as the single module in the preceding section. The states of operation of the modules and transition probabilities between states are defined. Then the expression for the expected tracks L waiting and the divergence condition can be developed.

b. State (n, a')

The state (n, a') is defined as the state where module M_1 is performing the detection and position determination functions in a region of the display necessary to process a track (t_1) , and module M_2 is waiting to process this track when sent from module M_1 , and n tracks are waiting to be detected and/or processed by module M_1 . This can be represented by the following diagram:



When module M_1 detects and determines the position of track (t_1) , it then sends this track to module M_2 . Module M_1 then returns to detect and process (position determination) another track.

It is assumed that the transition out of state (n, a') into state (n, b') occurs with an average rate μ_1 .

c. State (n, b')

The state (n, b') is defined as the state where module M_1 is processing a track and module M_2 is processing the previous track processed by module M_1 , and n tracks are waiting to be processed by module M_1 . This can be represented by the following diagram:



Module M_2 is now transforming the track (t_1) from the position to the velocity information state. When it completes this transformation, track (t_1) is sent to the identification module M_4 . This means the modules return to the state (n, a').

It is assumed that the transition from state (n, b') to state (n, a') can occur with an average rate μ_2 .

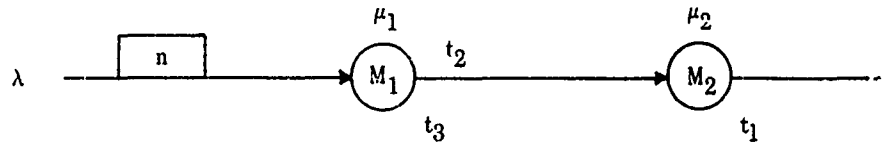
Another transition possibility exists in state (n, b'). This is the possibility of module M_1 completing its processing on a track (t_2) but module M_2 has not completed processing (t_1). Module M_1 cannot send track (t_2) to module M_2 until M_2 is finished processing (t_1). This restriction is due to the radar noise conditions which may exist. These noise conditions require module M_1 to continue processing the track until module M_2 is ready to accept it. This is the blocking effect mentioned earlier.

It is assumed that the transition out of state (n, b') to state (n, c') can occur with an average rate μ_1 .

d. State (n, c')

The state (n, c') is defined as the state where module M_1 has completed processing track (t_2) but module M_2 is still processing track t_2 , and n tracks are waiting to be processed by module M_1 .

This can be represented by the following diagram:



When module M_2 completes processing track t_1 it can then accept track t_2 from module M_1 . At this time module M_1 is free to process track t_3 . The modules are now in state (n, b') defined above.

It is assumed that the transition from state (n, c') to state (n, b') occurs with an average rate μ_2 .

Therefore, the following module state transitions are possible:

$$a' \xrightarrow{\mu_1 dt} b'$$

$$b' \xrightarrow{\mu_2 dt} a'$$

$$b' \xrightarrow{\mu_1 dt} c'$$

$$c' \xrightarrow{\mu_2 dt} b'$$

The balance equations for modules M_1 and M_2 can now be written from equation 34. They become:

$$\frac{d}{dt} P(n, a') = P(n-1, a') - (\lambda + \mu_1) P(n, a') + \mu_2 P(n, b')$$

$$\frac{d}{dt} P(n, b') = \mu_1 P(n+1, a') + \lambda P(n-1, b') - (\lambda + \mu_1 + \mu_2) P(n, b')$$

$$+ \mu_2 P(n, c')$$

$$\frac{d}{dt} P(n, c') = \mu_1 P(n+1, b') + \lambda P(n-1, c') - (\lambda + \mu_2) P(n, c'). \quad (44)$$

For purposes of comparing the two-module operation with the single-module operation of the previous section, we now let $\mu_1 = \mu_2 = \mu$. The above equations now become:

$$\begin{aligned}
\frac{d}{dt} P(n, a') &= P(n-1, a') - (1 + \frac{1}{\rho}) P(n, a') + \frac{1}{\rho} P(n, b') \\
\frac{d}{dt} P(n, b') &= \frac{1}{\rho} P(n+1, a') + P(n-1, b') - (1 + \frac{2}{\rho}) P(n, b') \\
&+ \frac{1}{\rho} P(n, c') \\
\frac{d}{dt} P(n, c') &= \frac{1}{\rho} P(n+1, b') + P(n-1, c') - (1 + \frac{1}{\rho}) P(n, c')
\end{aligned} \tag{45}$$

where

$$\rho = \frac{\lambda}{\mu}.$$

The steady state probabilities become, for $n \geq 1$,

$$\begin{aligned}
P(n-1, a') - (1 + \frac{1}{\rho}) P(n, a') + \frac{1}{\rho} P(n, b') &= 0 \\
\frac{1}{\rho} P(n+1, a') + P(n-1, b') - (1 + \frac{2}{\rho}) P(n, b') + \frac{1}{\rho} P(n, c') &= 0 \\
\frac{1}{\rho} P(n+1, b') + P(n-1, c') - (1 + \frac{1}{\rho}) P(n, c') &= 0
\end{aligned} \tag{46}$$

For $n = 0$, the initial condition equations become:

$$\begin{aligned}
-(1 + \frac{1}{\rho}) P(0, a') + \frac{1}{\rho} P(0, b') &= 0 \\
\frac{1}{\rho} P(1, a') - (1 + \frac{2}{\rho}) P(0, b') + \frac{1}{\rho} P(0, c') &= 0 \\
\frac{1}{\rho} P(1, b') - (1 + \frac{1}{\rho}) P(0, c') &= 0.
\end{aligned} \tag{47}$$

The determinantal equation for equation 46 becomes:

$$\begin{vmatrix} 1 - (1 + \frac{1}{\rho}) E & \frac{1}{\rho} E & 0 \\ \frac{1}{\rho} E^2 & 1 - (1 + \frac{2}{\rho}) E & \frac{1}{\rho} E \\ 0 & \frac{1}{\rho} E^2 & 1 - (1 + \frac{1}{\rho}) E \end{vmatrix} P_i'(n) = 0 \quad (18)$$

where $P_i'(n)$ represents any of the three steady state probabilities

$$i = (a', b', c').$$

The roots of the above determinantal equation are given in reference (13) and become:

$$\begin{aligned} r_1 &= 1 & r_3 &= 1/4\rho \left[(\rho + 3) - \sqrt{\rho^2 + 6\rho + 1} \right] \\ r_2 &= \rho/\rho + 1 & r_4 &= 1/4\rho \left[(\rho + 3) + \sqrt{\rho^2 + 6\rho + 1} \right] \end{aligned} \quad (49)$$

The solutions of the $P_i'(n)$'s become:

$$P_i'(n) = \sum_{j=1}^4 c_{ij} r_j^n \quad i = 1, 2, 3 \quad (50)$$

The constants c_{ij} are determined from equation 47.

The mean number of tracks L which can be expected to be waiting to be detected by module M_1 becomes:

$$L = \sum_{n=0}^{\infty} (n+1) P_i'(n). \quad (51)$$

The condition which assures that the roots will not be greater than one becomes:

$$\rho = 2/3 \quad (\text{Two Modules}). \quad (52)$$

This assures that the mean number of tracks (L) will reach a steady state and not diverge.

We now can compare the two-module M_1, M_2 operation with the single-module M_1 operation of the last section. For example, let $\rho_1 = \rho_2 = \rho$ in equation 43. This means that for the single-module operation the convergence condition is satisfied when:

$$\rho = 1/2 \quad (\text{Single Module}). \quad (53)$$

Thus, we see that the addition of module M_2 has increased the input rate capabilities but these capabilities can still be limited by the blocking effect caused by radar noise conditions. When radar noise conditions are low, such blocking would not occur and the queue in front of module M_2 could be permitted to grow.

This means that the queues and processing rates of modules M_1 and M_2 can be analyzed separately. The input rate to module M_1 would then be limited by the condition $\rho \leq 1$ which is well known in queuing theory. Such analyses have been carried out in the literature.

One of the indirect relationships which is very important in this context can involve the track monitor module along with the identification and weapons direction modules in a backward flow relationship. The track monitor module must, as specified, monitor tracks on a continuing basis, i.e., it does not actually complete responsibility for a track until that track becomes classified outside the sphere of interest of the system. Such classification can occur if the track departs from the surveillance sector, if it has been permanently identified as a friendly, or if it is a hostile already under attack. The operations of the track monitor module actually depend on the detailed operations of the succeeding modules. This difficulty was circumvented in the preceding section by dividing the track monitor into two sub-nodes, one which performed the initial monitoring on each track and one which assumed responsibility for monitoring on a continuing basis. This removes the portion of the track monitor which depends on succeeding decisions from the direct line of essential operations discussed here.

For the two-module case, the blocking effect can be treated analytically as shown above. When blocking effects occur involving a number of modules (men) in an organization, such analytical techniques become impossible and analysis must be carried out via simulation techniques. ⁽²⁰⁾

3. Conclusions

In conclusion, blocking effects represent the relationships which can influence the input processing rate capabilities of the system indirectly. If the sensitivity analysis treats the decision making rates of the modules separately and does not consider these blocking relationships between modules, such effects will not be related to the system performance. Also, when developing the processing requirements of the modules to minimize the track delay loss functions, the system must be treated as a whole. This was discussed in section VII.

The decision delays of modules M_4 , M_5 , and M_6 all influence the rate at which module M_2 can complete processing of the tracks. The blocking effect that module M_2 has on module M_1 then determines the input rate capabilities of the system. The number of tracks L waiting to be processed gives an indirect measure of the decision time delays of the modules and the blocking effects due to noise conditions (Figure 18).

The sensitivity analysis then requires that a loss function be developed for this waiting line L in the same manner as decision error losses were developed in the previous section. This would involve the establishment of track classes and priority functions before such a loss function could be developed for the track delays.

The sensitivity analysis is complete when the decision delay, error, and waiting line loss functions are developed and related to the system criterion C_0 . This is shown by the dotted vectors in Figure 18. Finally, the optimum goals or time constraints and track priority functions for each module's operation are changed in a manner which minimizes the overall system criterion loss function. It was stated in section VII that such changes would occur on-line via the control modules by knowledge of the sensitivities of module operations to other module operations and finally to the system criterion.

Two alternatives exist, when blocking effects and queues reduce the effective operation of the transformation modules M_1 through M_6 , and the system criterion cannot be achieved by establishment of module goals and simple first come-first serve sequencing disciplines. These are:

- (1) Introduce parallel operation
- (2) Utilize a more effective sequence discipline.

Parallel operation would involve the addition of another channel of transformation modules. Parallel operation will increase the system's processing rate without reducing the decision times of the modules. Parallel operation is well understood and will not be discussed.

The problem of what to do in the case of system overloads always arises in command and control systems. This problem also emphasizes the second alternative. This is the introduction of a more effective sequence discipline for the tracks. This means the possibilities of system control flexibility, by the higher echelon modules, must be exploited.

IX. SYSTEM CONTROL MODELS

The final step in the system analysis procedure is the development of any control flexibility which can be utilized in case of system overloads (large L). If precedence relations of the system prevent such flexibility then the only thing to do is add more capacity (additional module channels).

The introduction of a more effective sequencing discipline reduces the queue of a channel in an optimum manner and must be cognizant of the blocking effects described in the last section. This means that some sort of prediction and control is necessary and the modules also have to be treated as a whole (not separately).

The addition of the senior surveillance, identification, weapons and battle staff control modules M_s , M_I , M_w , and M_B , in Figure 15 allows such control of the lower echelon modules M_1 , M_2 , M_3 , M_4 , M_5 , and M_6 . These control modules now permit the lower echelon modules to perform the transformation processes on the tracks and free them from the ordering function of determining which track to process. Therefore, while the lower echelon performs the processing required for transforming the information states of the tracks, the control modules can perform the data collection, predictions, and computation necessary for determining how the modules should perform as a whole.

Data collection for such control purposes would involve collecting the statistics of the past performance of the system. This involves the input track characteristics, input noise statistics on radar noise, flight plans, and weapon status, the performance of the decision element (man), and the resulting system performance. From this, the errors and delays of the decision element could be predicted for future similar situations.

Such future predictions would then involve knowledge of the noise conditions from the material inputs modules M_m , M_n , and M_o , (Figure 15). Since all noise conditions cannot be known by these modules, the system can be controlled on-line only by knowledge of the known or predictable noise characteristics. Figure 13 showed some typical noise characteristics.

In this section knowledge of a selected set of these noise conditions will be assumed and an on-line control or sequencing model will be developed for the surveil-

lance and identification modules. It will also be seen that such sequencing or combinatorial models yield insight into the information transformation sequences and existing decision flexibilities of the modules. (Combinatorial models are utilized frequently in industrial control systems for investigating the sequencing of a number of jobs through a number of machines in some defined optimum manner). In this section we will consider the modules as the machines and the processing of the tracks as the jobs.

Many sequence disciplines are possible for performing the track transformations by the surveillance and identification groups. Some of these are:

- (1) First come-first serve
- (2) Pre-emptive priority
- (3) Random selection
- (4) Shortest processing time
- (5) Dynamic priority rules.

These sequencing disciplines are well understood and are discussed in the literature.

Frequently, however, in large scale systems such as industrial control systems, and in command and control systems such as SAGE, each group of modules cannot have a sequence discipline that is independent of the other group operations. That is, the system must be operated (or controlled) as a whole to achieve the system criterion. This was discussed in the sensitivity analysis section of the report when the loss functions of each mode of operation (group) were determined by analyzing the system as a whole.

To demonstrate such intergroup dependence, and the on-line control of the modules, a sub-goal of the system criterion will be considered. The goal of processing n tracks through the surveillance and identification nodes of operation in minimum time will be chosen. The control and prediction required to achieve this minimum time goal will be investigated for the two following cases:

- (1) Two-Group Sequence Discipline
- (2) Multiple-Module Sequence Discipline.

The two-group discipline involves sequencing n tracks through the surveillance and identification groups. The multiple module discipline involves sequencing n tracks through the four transformation modules M_1 , M_2 , M_3 , and M_4 .

A. TWO-GROUP SEQUENCE MODEL

The following presents the goal and assumptions of the models:

1. The goal is defined as that of processing n given tracks in queue through the surveillance and identification groups in minimum time.
2. No priority rules exist for any tracks. That is, all tracks exist in an equal priority region.
3. The radar noise conditions in the region of each track are known or predictable. This information is obtained from the radar status module M_m and sent to the control module M_s (Figure 15).
4. Flight plan status noise conditions in the region of each track are known. This refers to the absence of flight plans or extraneous (more than one) flight plans in the track region. This information is obtained from the flight plan status module M_n and sent to the control module M_I .
5. The control module M_s can predict or specify transformation times of modules M_1 , M_2 , and M_3 , for processing a track, given the noise conditions in the region of a track. These times are different for each track (different noise conditions).
6. The control module M_I can predict or specify the identification time of module M_4 , given the flight plan noise condition in the region of a track. These times are different for each track.
7. The radar and flight plan noise conditions in the region of each track are different but remain fixed for the duration of sequencing of the n tracks. This means that if a track is processed first or last, the time of processing is the same.

The above minimum time goal and assumptions lead to the following sequencing model for the two-group operation:

Given n tracks, with known processing times in the two groups, determine the sequence that will process the n tracks through the two groups in minimum time. In combinatorial analysis, this is the familiar n job - two machine problem.

The number of possible sequences of n tracks through two groups becomes $(n!)^M = 2$. The constraint that each track must proceed in order through the two groups reduces the possible sequences to $(n!)$.

ρ_i = Expected time to process track i by the surveillance group.

β_i = Expected time to process track i by the identification group.

n = Number of tracks existing in surveillance group to be processed.

T = Total time to process n tracks through the groups $(i_1, i_2, i_3, \dots, i_n)$.

x_i = Idle time of the identification module M_i from the end of track $(i-1)$ to the beginning of track (i) .

$\{S\}$ = An arbitrary sequence of tracks $(i_1, i_2, i_3, \dots, i_n)$.

$$T = \sum_{i=1}^n \beta_i + \sum_{i=1}^n x_i. \quad (54)$$

We then wish to find a sequence of tracks that will minimize T . The first term of this expression is always a constant.

$$\sum_{i=1}^n \beta_i = \text{constant} \quad (55)$$

Therefore, the problem becomes that of finding the sequence for which

$$\sum_{i=1}^n x_i \rightarrow \text{minimum}. \quad (56)$$

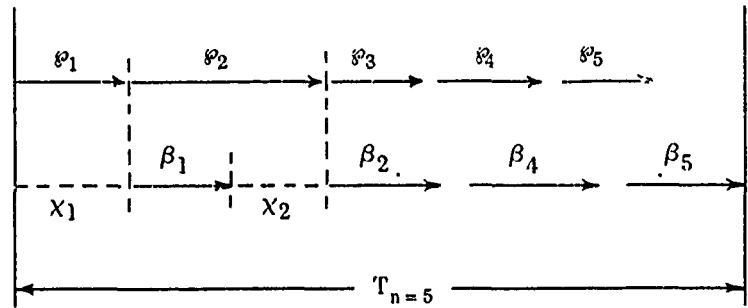
We first determine an expression for the idle time of an arbitrary sequence S and then determine an algorithm which will determine the sequence with the minimum idle time $\{S\}_{\min}$ and thus the minimum time T_{\min} to process the tracks through the surveillance and identification groups.

Figure 23 shows a Gantt or sequencing chart for the surveillance and identification groups for five tracks, where

$$\begin{aligned} x_1 &= p_1 \\ x_2 &= p_1 + p_2 - \beta_1 - x_1 \quad \text{If } (p_1 + p_2 \geq x_1 + \beta_1) \\ &= 0 \quad \text{If } (p_1 + p_2 \leq x_1 + \beta_1). \end{aligned} \quad (57)$$

This expression for x_2 can also be written:

$$\begin{aligned} x_2 &= \text{Max} \left[\left(p_1 + p_2 - \beta_1 - x_1 \right), 0 \right] \\ &= \text{Max} \left[\left(\sum_{i=1}^2 p_i - \sum_{i=1}^1 \beta_i - \sum_{i=1}^1 x_i \right), 0 \right] \end{aligned} \quad (58)$$



- $\overrightarrow{\rho_i}$ = Processing Time in Surveillance Group
 $\overrightarrow{\beta_j}$ = Processing Time in Identification Group
 $\overrightarrow{x_k}$ = Idle Time of Identification Group

Figure 23. Sequencing Chart for Surveillance and Identification Groups

The total idle time for two tracks can then be written:

$$x_1 + x_2 = x_1 + \text{Max} \left[(\rho_1 + \rho_2 - \beta_1 - x_1), 0 \right]. \quad (59)$$

Then, if the second term is zero,

$$x_1 + x_2 = x_1. \quad (60)$$

If the second term is $(\rho_1 + \rho_2 - \beta_1 - x_1)$,

$$x_1 + x_2 = \rho_1 + \rho_2 - \beta_1. \quad (61)$$

Therefore, equation 59 can be written

$$\begin{aligned}
 x_1 + x_2 &= \text{Max} \left[(\rho_1 + \rho_2 - \beta_1), x_1 \right] \\
 &= \text{Max} \left[\left(\sum_{i=1}^2 \rho_i - \sum_{i=1}^1 \beta_i \right), x_1 \right].
 \end{aligned} \quad (62)$$

Then, for three tracks,

$$x_3 = \text{Max} \left[\left(\sum_{i=1}^3 \rho_i - \sum_{i=1}^2 \beta_i - \sum_{i=1}^2 x_i \right), 0 \right] \quad (63)$$

and

$$\sum_{i=1}^3 x_i = \text{Max} \left[\left(\sum_{i=1}^3 \rho_i - \sum_{i=1}^2 \beta_i \right), \left(\sum_{i=1}^2 \rho_i - \beta_i \right), \rho_i \right]. \quad (64)$$

Then, we let

$$D_n(S) = \sum_{i=1}^n x_i \quad (65)$$

$D_n(S)$ is a function of the particular sequence, and it can be written:

$$D_n(S) = \sum_{i=1}^n x_i = \max \left[\sum_{i=1}^n \rho_i - \sum_{i=1}^{n-1} \beta_i, \sum_{i=1}^{n-1} \rho_i - \sum_{i=1}^{n-2} \beta_i, \dots, \rho_1 \right]$$

$$D_n(S) = \max_{1 \leq u \leq n} \left[\sum_{i=1}^u \rho_i - \sum_{i=1}^{u-1} \beta_i \right]. \quad (66)$$

$D_n(S)$ becomes the function which is to be minimized by selection of the optimum sequence. This is a combinatorial problem since $D_n(S)$ is a function of a sequence of n tracks. It then remains to develop an algorithm for determining the optimum sequence for any arbitrary sequence. The derivation of such an algorithm is given in reference (18).

This algorithm minimizes the total time T and is obtained by the following sequencing rule:

Track j precedes track $j+1$ if

$$\text{Min}(\rho_j, \beta_{j+1}) < \text{Min}(\rho_{j+1}, \beta_j);$$

Either track precedes the other if

$$\text{Min}(\rho_j, \beta_{j+1}) = \text{Min}(\rho_{j+1}, \beta_j).$$

After the ρ_i 's and β_i 's have been determined, the procedure for the control module M_s to develop the optimum sequence becomes:

1. Find the smallest value

$$\text{Min}(\rho_i, \beta_i).$$

2. If the smallest value is an ρ_i , the corresponding track is ordered first.
3. If the smallest value is a β_i , the corresponding track is ordered last.
4. Each time a track is ordered, it is eliminated from further comparisons.
5. Continue the above four steps until each track has been ordered.

Consider the following case with five tracks with predicted processing times

ρ_i and β_i .

Track i	ρ_i	β_i
1	3	7
2	4	2
3	5	4
4	6	1
5	1	3

Following the above rule the optimum sequence for $T = T_{\min}$ becomes:

$$\{S_0\} = \{5, 1, 3, 2, 4\}.$$

It can be concluded that if the necessary information regarding the radar noise status from module M_m and flight plan status from module M_n can be collected, the surveillance module M_s can determine the minimum time sequence.

The identification group will process the tracks on a first come-first serve basis. The surveillance group then determines the sequence for both groups. Therefore, with a minimum time criterion, we see that the two groups must work in coordination with one another when such processing time predictions are possible.

Another factor should be noted from the sequencing diagram. This is in regard to the idle time of a group. After the minimum time sequence is determined, these idle times give the tracks which can be processed longer if the need arises without affecting the minimum time goal. Therefore, the control of transformation (or decision) times is also suggested by such sequencing models. Such time control may arise when the question arises whether or not to permit additional processing time for the sake of increased accuracy. We see here that in some cases such increased time will not affect the minimum time goal and possibly increase the accuracy.

B. MULTIPLE MODULE SEQUENCE DISCIPLINE

It was stated in the last section that a truly optimum sequence discipline must include each of the module transformation processes. The last section treated

the modules M_1 , M_2 , M_3 of the surveillance group as one node of operation. That is, the time delay of the surveillance group was assumed to be p_i for track i . This did not permit any kind of control of these modules. This procedure can then only be treated as a sub-optimum sequencing technique. However, as was seen, the computation required for this control was very simple.

To determine a true minimum time sequence, the modules M_1 , M_2 , M_3 , and M_4 have to be treated separately. That is, the track transformation times on each module should be utilized for developing a sequencing discipline to achieve the minimum time goal.

This means that if n tracks exist in queue in the surveillance group, there exist

$$(n!)^m \quad (n!)^{m-1}$$

possible sequences to process these tracks by the $m-1$ modules.

At the present time this general combinatorial problem has no analytical solution (such as the n track - 2 module case). Many techniques have been utilized to obtain near-optimal solutions. For the n track - m module case, with a minimum time goal, the computer program sequencing technique developed by E.S. Schwartz appears to be most applicable.⁽¹⁴⁾ In this section, this technique is adapted to the on-line control of the man-computer-display modules. It again gives insight into determining which transformation processes can be increased in time to obtain increased accuracy without affecting the minimum time goal.

The n track - m module problem posed by the SAGE system is built up by placing in parallel, identical transformation structures. Each transformation must be performed by corresponding modules. Such a structure is shown in Figure 24. Each structure represents one track in the SAGE system and n of these structures represents the n track - m module problem. The nodes in Figure 24 are the transformations on a track discussed in a previous section. The corresponding module required for this transformation is M_i , where:

M_1 - Track Initiator Module: Performs position transformation τ_1

M_2 - Track Monitor Module: Performs velocity-direction transformation τ_2

M_3 - Height Finding Module: Performs height transformation τ_3

M_4 - Identification Module: Performs identification transformation τ_4

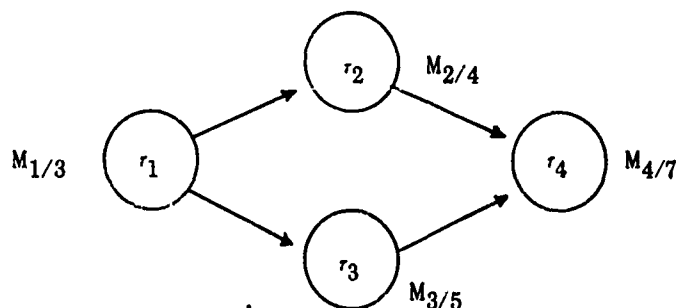


Figure 24 Typical Track Transformation Structure

The basic assumption as discussed earlier is that the expected time required for each module to perform its corresponding transformation is known by the senior control modules. These times, along with the module designation, are indicated above the proper transformation. Finally, a typical track transformation structure can be represented by Figure 24.

The arrows between the nodes represent the direct precedence relations of the transformations. That is, velocity-direction and height cannot be obtained before position. Identification cannot be performed before the velocity-direction and height transformations.

The problem posed by the minimum time goal can now be stated formally: Given n tracks with the above transformation structure, determine the sequence that identifies all tracks in minimum time. A three-track problem is represented in Figure 25. The transformations have been numbered for representation in an assignment matrix.

The assignment matrix is a rectangular alpha numeric array which contains all the information necessary to solve the sequencing problem. This assignment matrix can easily be derived from a first order precedence matrix, M . The first order precedence matrix is a square array containing the Boolean variables 0 and 1. If transformation τ_i is preceded by transformation τ_j , then the element $M(i, j)$ of the first order precedence matrix is 1; otherwise all elements of this matrix are 0. M will be of a lower triangular form if the transformations are numbered so that τ_i is preceded by τ_j only if j is less than i . It is only necessary to include first order relationships in the matrix M . That is, in scanning row i we are interested only in the column j for which τ_i is directly preceded by τ_j . Higher order relationships can then be obtained from M . The first order precedence matrix for the three-track SAGE problem is shown in Figure 26. Blanks replace the Boolean zeros in this figure.

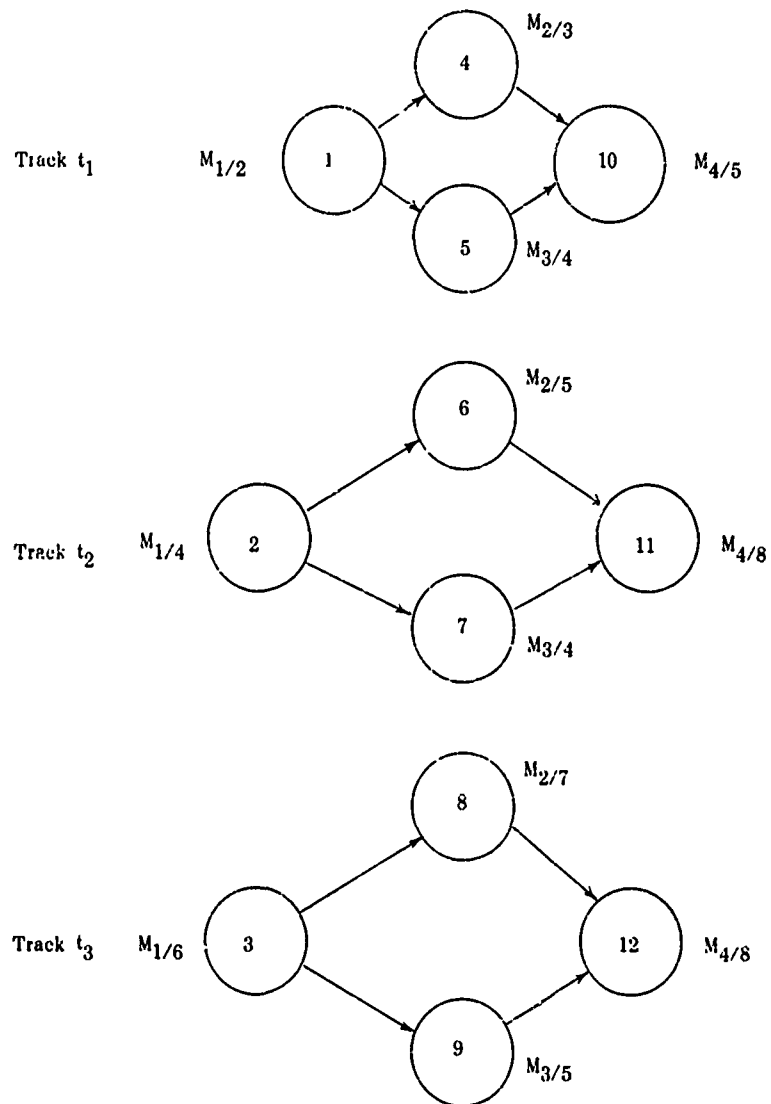


Figure 25. Three-Track Sequencing Problem

Higher order relationships are now obtained by Boolean multiplication and addition. These operations are indicated by the symbols \wedge and \vee , respectively, and are defined by the following relations:

$$A \wedge B = 1 \text{ if and only if } A = 1 \text{ and } B = 1$$

$$A \vee B = 1 \text{ if and only if either } A = 1, \text{ or } B = 1, \text{ or both are equal to } 1.$$

M =

	1	2	3	4	5	6	7	8	9	10	11	12
1												
2												
3												
4												
5												
6												
7												
8												
9												
10												
11												
12												

Figure 26. First Order Precedence Matrix for Three-Track Problem

We now proceed by raising the matrix M to higher and higher powers until we obtain a zero matrix. The multiplication involved in raising M to these powers is Boolean multiplication. This operation is exactly analogous to ordinary matrix multiplication with the operations of Boolean multiplication and addition defined above replacing ordinary multiplication and addition. Each power of M reveals higher order relationships among the modules until all precedence relations are exhausted. We now form the higher order precedence matrix, H , by taking the Boolean sum of these powers of M :

$$H = M \vee M^2 \vee M^3 \vee \dots \vee M^{p-1}, \text{ where } M^p = 0.$$

In the three-track problem we are considering $p = 3$ so that H involves only second order relations. Figure 27 is the precedence matrix, H , for the three-track problem.

In most of the problems examined in the course of development, the precedence matrix H can be written down directly without going through the process of writing the first order precedence matrix and raising to powers. The principal reason for this is that the SAGE system, as it is modeled here, involves only second order relations. In systems involving higher order precedence relations it is advantageous to follow this procedure.

The precedence matrix contains all necessary precedence restrictions on the problem. The assignment matrix is now constructed from the precedence matrix

H E

	1	2	3	4	5	6	7	8	9	10	11	12
1												
2												
3												
4												
5												
6												
7												
8												
9												
10												
11												
12												

Figure 27 Higher Order Precedence Matrix for Three-Track Problem

by left justifying (moving ones to the left) all rows of the precedence matrix, replacing the Boolean ones by the numbers of the corresponding modules, and indicating the processing module. The assignment matrix now contains all known information about the problem except the processing times; these are added to the assignment matrix as a column vector. The left justified matrix for a three-track problem is shown in Figure 28 and the assignment matrix in Figure 29. The assignment matrix now contains all the information about the problem.

The assignment matrix is an alphanumeric array in which each row represents a transformation τ_i . Beginning at the left of the array, integers representing transformations which precede a given transformation are entered in order of precedence. After all preceding transformations are listed, a letter designating the module M_i which performs the transformation is added. All other elements of the assignment matrix are zero or blank.

The final column of the matrix contains the processing time of each transformation. Topological considerations can also be included in the assignment matrix but they are not important for the SAGE system as we have modeled it here. The assignment matrix shown in Figure 29 now contains all the information about the problem.

As transformations are processed by the designated modules, the transformation designation and corresponding integers in the matrix are converted to zero. A transformation is said to become unbounded when no integers precede its module designation in the assignment matrix.

	1	2	3	4	5	6	7	8	9	10	11	12
1												
2												
3												
4												
5												
6												
7												
8												
9												
10												
11												
12												

Figure 28. Left Justified Precedence Matrix for Three-Track Problem

1	M ₁									2
2	M ₁									4
3	M ₁									6
4	1	M ₂								3
5	1	M ₂								4
6	2	M ₂								5
7	2	M ₂								4
8	3	M ₂								7
9	3	M ₂								5
10	1	4	5	M ₄						5
11	2	6	7	M ₄						8
12	3	8	9	M ₄						8

Figure 29. Assignment Matrix for Three-Track Problem

Sequencing then consists of scanning the assignment matrix for unbounded transformations, converting integers to zero, and repeating until the assignment matrix is reduced to zero and the $n = 3$ tracks have been processed.

Sequencing proceeds according to these rules:

- Rule 1. A transformation is assigned to its designated module in the assignment period in which it becomes unbounded.
- Rule 2. When two or more transformations are unbounded at the same assignment period, assign them for processing in the order of increasing processing time.

These rules describe a procedure of obtaining an optimum or near optimum minimum time sequence. The sequencing procedure minimizes the expected time to reach the next transformation assignment period. This procedure is not necessarily equivalent to total optimization although it seems to come close in most cases. Schwartz⁽¹⁴⁾ has also noted that optimum or near optimum solutions have been obtained in all cases he has tried.

In the problem shown in Figure 25, the sequencing procedure does result in an optimal solution. Figure 30 shows a Gantt chart of this solution and two alternate, worse solutions. Figure 31 is a problem in which the sequencing procedure does not produce an optimal solution. Figure 32 shows Gantt charts for the solutions of this problem.

C. CONCLUSION

The advantages of the sequencing procedure are two-fold. First, the precedence restrictions insure that all solutions will be feasible. Second, since the total sequence is not analyzed at each stage, the procedure produces a solution rapidly. This second advantage is an important one when considering man as the system controller of such transformation sequences.

This section also pointed out the need for intergroup communication for the purpose of developing optimum procedures for the lower echelon modules. That is, two groups in the organization cannot perform with independent goals. These goals must be compatible and assure the achievement of the overall system criterion. Figure 18 showed the intergroup goal communication paths between the control modules. In this section, the communication requirement between modules M_S and M_I to achieve the minimum time goal, was the flight plan status information.

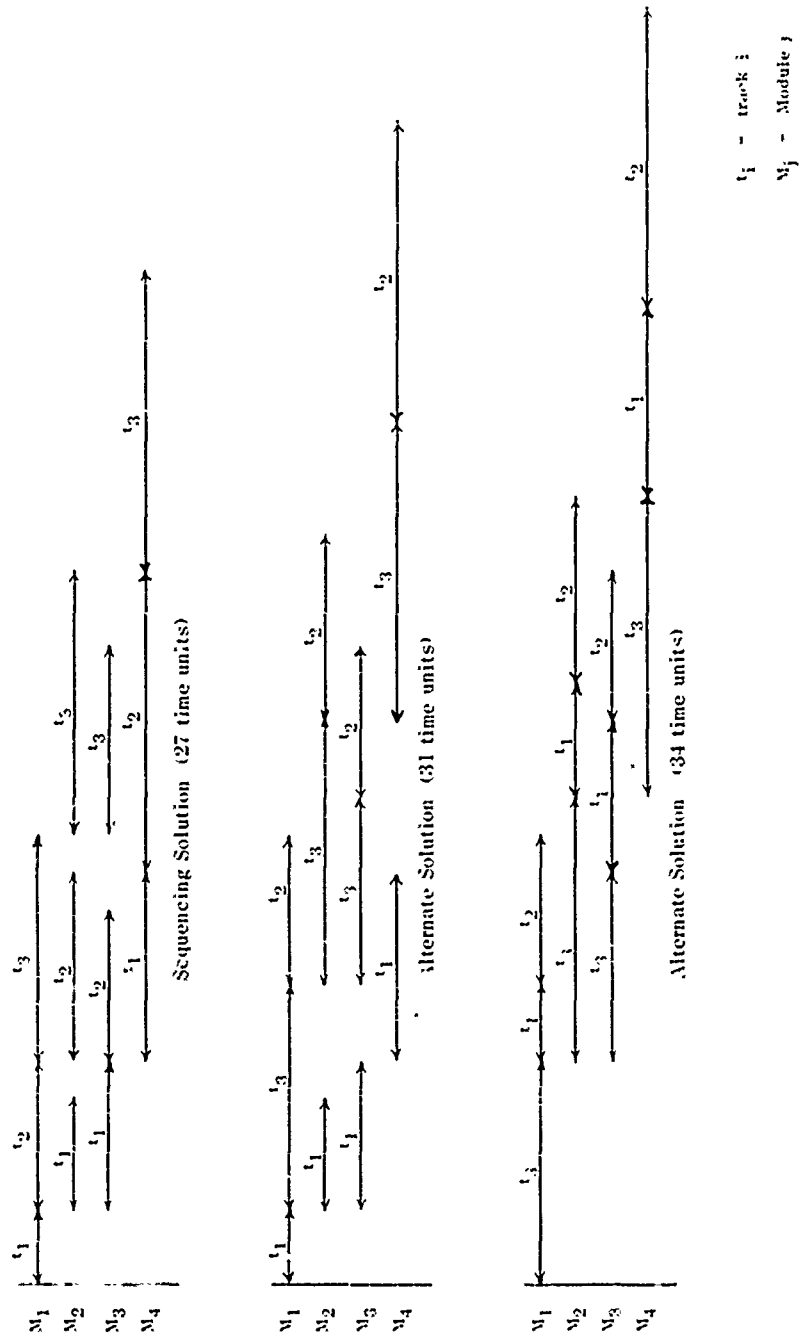


Figure 30. Gantt Charts of Solutions to Problem in Figure 25.

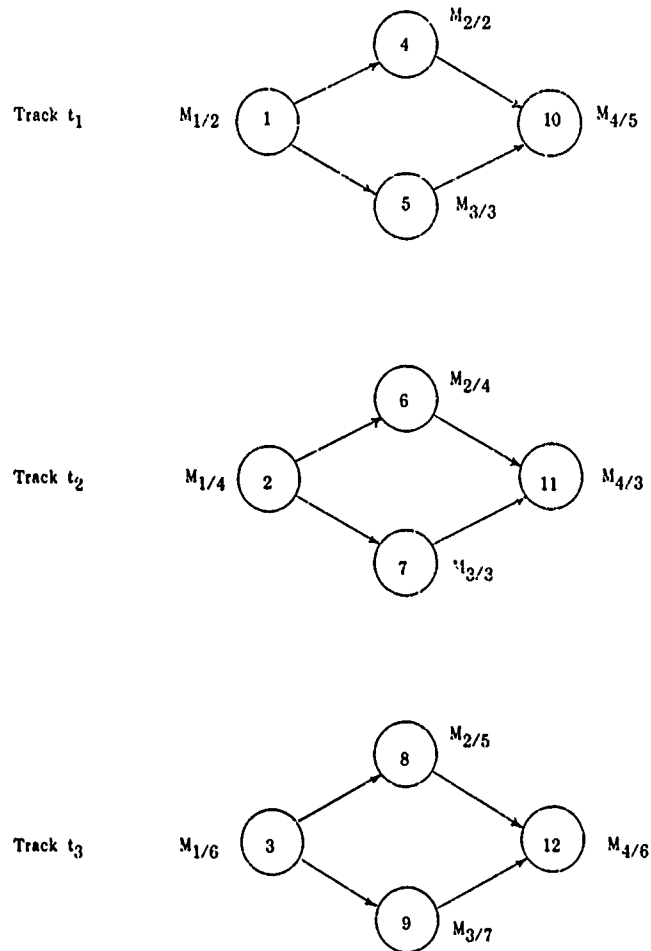


Figure 31. Three-Track Problem to Illustrate Non-Optimal Sequencing Solution

In order that the upper echelons perform effective control of the lower echelons in the SAGE system it can be seen that noise data communication and prediction for specification of the transformation times is at least necessary. This, of course, is a problem and must be solved by other means.

One such method already exists in SAGE. This is with regard to the displaying of radar tracking figures of merit on each track to the men (transformation modules). When radar noise conditions on a track are poor, fair or good, for automatic tracking by the computer, these noise conditions are displayed to the man. This then gives him a measure of confidence regarding the position, velocity, and

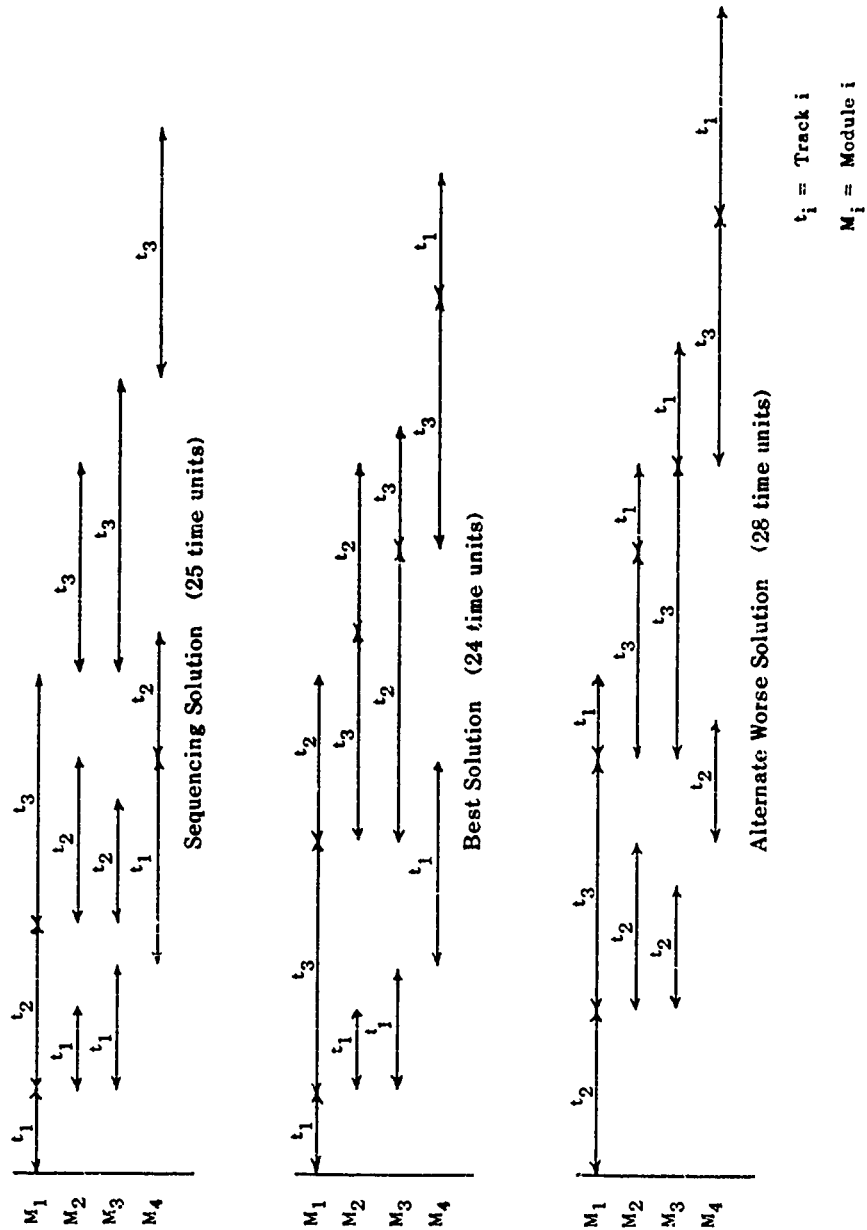


Figure 32. Gantt Charts of Solutions to Problem in Figure 26

direction track coordinate accuracies. If tracking merit is poor, he may spend more time monitoring the track through noise. More time may also be spent making the other track decisions with regard to identification and weapons assignment and control. When tracking merit is good, he may accept the tracking information, make his decision and begin processing another track.

Therefore, the tracking merit indication from the computer performs the track time-error control of the lower echelon modules in a limited indirect manner.

1. Man as a Controller:

To produce a true optimum sequence of track transformations through the system modules, all possible sequences may have to be examined. When the number of tracks is large, the time required to examine all sequences would probably outweigh the time saving of finding the minimum time sequence.

Therefore, sub-optimum sequencing techniques, in the combinatorial sense, cannot be considered sub-optimum when considering how fast men in an organization can collect, predict, and control such lower echelon decision and information handling functions. The technique by Schwartz appears to be close to this practical trade-off required for effective on-line control of men.

Another control variable that was pointed out in the beginning of the section was the time spent on performing a transformation (decision function) by the decision element. From the sequencing diagrams, the idle times between transformations indicate which transformations can be extended in time, if the need arises, without affecting the minimum time goal. Such a need may arise if the expected time to perform a transformation was found to be longer by the module because of changing noise conditions. In this case the senior surveillance module has to consider the consequences of letting a module continue processing the track in question or switching to another track. If an idle time exists, then such a continuation would not affect the total sequence. If such an idle time does not exist, the consequences have to be considered between accepting the module's decision early (before enough information has been acquired) or delaying the remaining track transformations to obtain a more accurate decision.

2. System Analysis Considerations:

The implications of combinatorial techniques at the system analysis level are apparent. Such models give insight into the decision making functions of

the men when working towards a common system criterion. For off-line analysis of the system, such models can be utilized for comparison purposes after the system has performed.

The second implication is the on-line control of the system. If a computer can store past statistics on the system and gather the existing noise conditions in a region of a track, then it should be able to perform a distributing function among the men. That is, it should be able to determine when, which man (module), should perform what function, on which track.

X. REFINED FUNCTIONS AND ORGANIZATION

The final step in the systems analysis procedure is the refinement of the module functional blocks and organizational structure. This structure was shown in Figure 15. This final step is performed after all on-line control flexibility has been exploited to meet the system criterion.

The sensitivity analysis presented the loss rate of the system due to decision errors and delays. It also gave the processing rate requirements of the track initiator and track monitor modules. The most obvious organizational change was seen to be that of introducing parallel channels for these module functions. This was necessary not only for high input rates and radar noise conditions, but also due to the fact that the remaining system modules were dependent upon the track monitor module.

The control analysis further exploited the possibilities of increased processing rates by the lower echelons, of equal priority tracks, by investigating their combinatorial features. It was shown that this required on-line information of noise conditions by the upper echelons. This requirement was possible to eliminate by allowing the lower echelons to control their own decision times by displaying noise condition information to them directly, (radar tracking figures of merit).

Therefore, the obvious organizational change for reducing the loss rate of the system is to develop priority functions on information known, or estimated in connection with each possible track. This type of priority rule is particularly important in the weapons assignment group.

The Bayesian formulations assumed priority rules to govern the operation of the various groups. A priority rule, being a deterministic function on the estimated characteristics of a track, has the property that the generally changing priority figure for a given track may be continually computed as the track proceeds through the system. It would be a simple matter to provide for continual automatic indication of the track with the highest priority at any particular module. In this scheme, the higher echelon would not be responsible for directing the individual track choices of the lower echelons.

The higher echelon would be responsible, however, for assuming that the particular priority functions being calculated in the supporting computer of the lower echelon are appropriate for the overall statistics and weapons status prevailing at any particular time.

The priority rule - lower echelon - upper echelon relationships are slightly different for the track initiator module. Here, priority is not determined by mathematical calculation, but by approximate screen position as indicated by a mask. It is the responsibility of the upper echelon to assure that the correct mask for the prevailing overall statistics and weapons status is being used at any particular time.

Therefore, it can be seen that further refinement of a command and control system lies in the ability to utilize past performance statistics to aid future lower echelon performance. For high speed operation, means must be sought for displaying such statistical information to the lower echelons to decrease their delays, errors, and improve their ordering.

The system becomes adaptive in nature when the resulting output performance of the system changes the a priori probability distribution for the next operation of the system. This is shown by the dotted line in Figure 1.

XI. CONCLUSIONS

The study utilized the SAGE system as a vehicle for developing a generalized systems analysis approach to command and control systems and for developing general conclusions which could be applied to the analysis of other command and control systems. The general conclusions of the report and recommendations for further study are presented below.

1. The purpose of the centralized command and control system concept is the accurate and timely detection of and reaction to information patterns. In SAGE, these information patterns related to the information states of airborne tracks and the information state of a threat such as an enemy raid.

2. The system analysis procedure emphasized relating the sensitivities of the decision delays and errors of the decision makers to the delays and errors of these information patterns.

3. The physical material handling and commodity formulation processes of an industrial control system present good abstract models for analyzing these information patterns and the information handling and decision making processes of a command and control system.

4. These abstract models were also useful for developing the errors and delays of the decision makers with regard to the information patterns. These patterns were the information states of tracks in the SAGE system.

5. The most promising use of the industrial control system analogies for future command and control system analysis is in the field of combinatorial analysis (or sequencing analysis). Future study should pursue how the decision sequences throughout an organization can be modeled and optimized by such analysis techniques.

6. The information inputs to a command and control system may be many dissimilar types of information. However, the noise variables on these inputs which affect the errors and delays of the decision makers are quite similar. In SAGE, these noise variables were characterized by absent, delayed erroneous, and extraneous information inputs. These variables related to the radar returns, flight plan inputs, and, to some degree, for the weapon status information inputs.

7. The decision makers of the SAGE system could be modeled in a homogenous fashion because of the similarity in the input information noise variables. That is, each decision maker could be modeled by a decision delay and error variable(s).

8. Because of the probabilistic nature of the noise variables, the mathematical modeling of the decision makers had to be probabilistic in two ways. First, the decision error probabilities were developed by Bayesian formulations in most cases. Second, the decision delays were modeled by Markov processes.

9. The Bayesian formulation indicated that any higher echelon control of the errors of the lower echelon decision makers would require knowledge of the a priori input information noise distributions and the conditional probability distributions of these decision makers. When such statistics are unavailable, knowledge of the decision makers' time delays in various noise conditions can be used for control.

10. The decision errors of each decision maker in the SAGE system were developed as functions of input information noise conditions and their information processing time (decision time delays). Further systems analysis requires determination of the actual decision maker's time-error relationships for various noise conditions on radar returns, flight plans, and weapon status information. Such relationships are necessary for any optimization of a command and control system organizational structure. This was shown to be especially true in the SAGE system where the system criterion was a function of both decision time delays and errors of each decision maker.

11. Each decision maker in SAGE had two conflicting goals. The first was an early decision. The second was an accurate decision on each track. The determination of the optimum trade-offs between these goals was further complicated by a changing track pattern environment. This meant that the added decision of which track or tracks to process affected these goals.

The Bayesian formulation stressed that there are only two kinds of losses associated with the SAGE system: the failure to intercept hostiles and the successful interception of friendlies. That is to say, the "time-accuracy trade-off" should be resolved by relating both factors to these two losses, rather than by attempting to associate losses with time or inaccuracy as such.

The Bayesian formulation showed that the time factor becomes related to the losses associated with the weapons direction - interception node through the defined probability density function of the coordinates of identified hostiles at the time

such tracks initiate with the weapons director. This probability density, in turn, influences the rate of loss of the system in an exceedingly complex manner which must be determined by simulation of the interception process.

These problems suggest areas for future decision theory work to determine the optimum time, error, and track order relationships in a changing track pattern under various noise conditions. Such information pattern changing problems of the environment, which complicate the decision processes, would be typical of a command and control system.

12. It was shown that direct and indirect relationships can exist between the decision makers of a command and control system. The errors of one decision maker can directly affect the errors of succeeding decision makers. In addition, time delays may indirectly affect the errors of the decision makers due to time constraints.

In SAGE, the identification decision process may require additional time due to noise conditions on a track. This time delay decreased the time available for weapon decision processes. If the track is hostile, this decreased time may result in ineffective weapon selection and assignment (weapon errors).

Further study of such direct and indirect decision relationships throughout command and control system organizations is required.

Digraph and matrix analysis techniques appear to be promising. In SAGE, such techniques were limited due to cyclic and various levels of relationships existing. Further study should investigate methods of detecting cyclic relationships from known direct relationships. In addition, methods of treating weak as well as strong indirect relationships between the decision makers must be determined.

13. The sensitivity analysis is the most difficult problem facing the mathematical analysis of command and control systems. Before system organizational changes can be made, the sensitivity of each decision maker's errors and delays to the system criterion must be developed.

It was shown in SAGE, that the following three problems complicated such sensitivity analysis: (1) Decision errors and delays must be treated in a probabilistic manner; (2) Indirect relationships may complicate developing direct sensitivity relationships to the system criterion; and (3) The complexity of the weapons assignment functions directly affect the track priority functions and processing times of the other decision makers in SAGE.

The report presented two models which demonstrated the complexity of such sensitivity analysis for relating errors and delays to the system criterion. One model utilized Bayesian formulations for developing the loss rate of the system due to the errors of the decision makers. The other model utilized Markov processes for developing processing requirement for the decision makers. The requirements assured that tracks entering the system would be processed and not delayed. The loss functions for such delays require further investigation of track priority functions. These can only be determined by further consideration of weapon assignment models for the SAGE system.

14. Combinatorial or sequencing analysis appears to be a promising area for future modeling and analysis of the human decision making sequences which occur in command and control systems. In the analysis of the SAGE system, these analysis techniques suggested methods by which to model the higher echelon on-line control of the lower echelon decision makers. The techniques also suggested ways in which to evaluate and relate the sensitivities of the decision sequences of the numerous decision makers to the system criterion.

FUTURE STUDY

Future modeling and analysis of command and control systems should be in the following areas:

- (1) Decision theory. Bayesian formulations are needed for the decision maker's error and time delay relationships under various input information noise conditions and varying information patterns such as target raids.
- (2) Digraph techniques. For development of systematic methods of detecting indirect time delay and error relationships between the decision makers from known direct relationships. These methods should give the system analyst ways of detecting weak, strong, and cyclic relationships.
- (3) Sensitivity analysis. For developing analysis techniques of relating the sensitivity of decision time delay and error variables of one decision maker to the other decision makers and finally to the system criterion. Simulation techniques are a requirement in the SAGE system, but there are two distinctly different types of simulations necessary. On the one hand the surveillance and identification functions, with their critical dependence on

human factors, are not amenable to simulation by computer. If it were possible to define the logic of the human beings in terms realizable on a computer, then the human beings would not be necessary. Simulation of the surveillance and identification nodes must contemplate recording the reactions of these nodes to known tracks and noise conditions in order to determine the mathematical characteristics defined in the sensitivity analysis.

These factors are necessary as inputs to the simulation of the weapons direction-interception node, which may be carried out on a computer, since the dependence in this case on human factors is slight (assuming that statistical data on the results of interception has been gathered from combat simulations). Since this node is the terminating point of SAGE, it can be conveniently studied in isolation from the rest of the system. These studies should develop the decision time delay-error relationships of the weapon selection and assignment functions.

- (4) Combinatorial analysis. For modeling and analyzing the higher echelon on-line control of the lower echelon decision making processes. These analysis techniques should also complement the sensitivity analysis by evaluating the decision sequences of the organization in an off-line analysis fashion. Such analysis would indicate optimum decision sequences and also decision time delays for gathering information.

Future combinations of combinatorial, sensitivity, and mathematical programming analysis techniques should lead to methods of determining the optimum allocation of information handling and decision functions among the decision makers in a command and control system organization.

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<p>Rome Air Development Center, Griffiss AF Base, NY Rpt No. RADG-TDR-62-612, COMMAND AND CONTROL SYSTEMS ANALYSIS, Final Report, Dec 62, 142 pp Incl illus</p> <p>Unclassified Report</p> <p>Results are described of a study utilizing the SAGE system as a vehicle for developing a generalized systems analysis approach to command and control systems and for developing general conclusions which could be applied to the analysis of other command and control systems. This report contains information from studies in such areas as: 1) Abstract Models of Functional Blocks 2) System Variables, Goals, and Indirect Relationships 3) System Sensitivity Analysis 4) System Control Models and 5) Recommended Functions and Organization. Recommendations for further study are also included. Some recommended areas are: 1) Decision Theory 2) Digraph Techniques 3) Sensitivity Analysis and 4) Combinatorial Analysis.</p>	<p>Operations Research Statistical Analysis Digital Computers Game Theory Project 5581 Task 558102 Cont AF30(602)-2752 Honeywell Military Products, Minneapolis, Minn. Secondary Report No. 1539- TR1 In ASTIA collection</p>	<p>Rome Air Development Center, Griffiss AF Base, NY Rpt No. RADG-TDR-62-612, COMMAND AND CONTROL SYSTEMS ANALYSIS, Final Report, Dec 62, 142 pp Incl illus</p> <p>Unclassified Report</p> <p>Results are described of a study utilizing the SAGE system as a vehicle for developing a generalized systems analysis approach to command and control systems and for developing general conclusions which could be applied to the analysis of other command and control systems. This report contains information from studies in such areas as: 1) Abstract Models of Functional Blocks 2) System Variables, Goals, and Indirect Relationships 3) System Sensitivity Analysis 4) System Control Models and 5) Recommended Functions and Organization. Recommendations for further study are also included. Some recommended areas are: 1) Decision Theory 2) Digraph Techniques 3) Sensitivity Analysis and 4) Combinatorial Analysis.</p>	<p>Operations Research Statistical Analysis Digital Computers Game Theory Project 5581 Task 558102 Cont AF30(602)-2752 Honeywell Military Products, Minneapolis, Minn. Secondary Report No. 1539- TR1 In ASTIA collection</p>
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